Syllabus for the Qualifying Exam in Algebra at Georgia State University

A. Groups:

Basic group theory, actions on groups and Sylow theorems.

B. Rings:

Basic ring theory, polynomial rings, Principal Ideal Domains, Unique Factorization Domains.

C. Fields:

Basic field theory, finite fields.

D. Galois theory:

Field extensions (finite, finitely generated, algebraic), algebraic closure, algebraically closed fields, splitting fields, normal extensions, separable extensions, the fundamental theorem of Galois theory, radical extensions, solvability with radicals of a polynomial equation.

E. Fundamental Symmetric Polynomials

Basic facts, the main theorem on symmetric polynomials, the fundamental theorem of algebra.

F. Module theory:

Basic theory of modules, direct sums, direct products, free modules, the theory of finitely generated modules over PIDs, invariant factors, elementary divisors, applications to finitely generated abelian groups.