◇ ◇ ◇ ◇ MATH 4442/6442: MODERN ALGEBRA II ◇ ◇ ◇ ◇
HOMEWORK SETS AND EXAMS

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Contents

Note. There are four (4) problems in each homework set. Math 6442 students need to do all 4 problems while Math 4442 students need to do any three (3) problems out the four. If a Math 4442 student submits all 4 problems, then one of the lowest score(s) is dropped. There is a bonus point for a Math 4442 student doing all 4 problems perfectly.

When solving homework problems, make sure that your arguments and computations are rigorous, accurate, and complete. Present your step-by-step work in your solutions/proofs.

There are three (3) PDF files for the homework sets and exams, one with the problems only, one with hints, and one with solutions. Links are available below.

PROBLEMS HINTS SOLUTIONS

Problem 1.1. Let R be a ring and $a, b \in R$.

(1) Expand $(a+b)^3$.
(2) Further assume that R is commutative. Expand $(a+b)^3$.

Hint. As an example, we illustrate how $(a+b)^2$ is done.
(1) $(a+b)^2 = (a+b)(a+b) = aa + ab + ba + bb = a^2 + ab + ba + b^2$.
(2) If R is commutative, then $(a+b)^2 = a^2+ab+ba+b^2 = a^2+ab+ab+b^2 = a^2+2ab+b^2$.

Problem 1.2. Let $R = \{a + b\sqrt[3]{2} + c\sqrt[3]{4} \mid a,\, b,\, c \in \mathbb{Z}\}$. Consider $(R,\, +,\, \cdot)$, with the usual addition and multiplication.

(1) Determine whether $(R,\, +)$ is an abelian group. No need to justify.
(2) Determine whether R is closed under multiplication. **Justify your claim.**
(3) Determine whether $(R,\, +,\, \cdot)$ is a ring. No need to justify.
(4) If $(R,\, +,\, \cdot)$ is a ring, determine whether it is a commutative ring.
(5) If $(R,\, +,\, \cdot)$ is a ring, determine whether it has unity.

Hint. Go by definition. Here we may use the fact (without proof) that, for all complex numbers (which include real numbers), associativity and distribution hold for addition and multiplication.

Problem 1.3. Let $S = \{a + b\sqrt[3]{12} + c\sqrt[3]{18} \mid a,\, b,\, c \in \mathbb{Z}\}$. Consider $(S,\, +,\, \cdot)$, with the usual addition and multiplication.

(1) Determine whether $(S,\, +)$ is an abelian group. No need to justify.
(2) Determine whether S is closed under multiplication. **Justify your claim.**
(3) Determine whether $(S,\, +,\, \cdot)$ is a ring. No need to justify.
(4) If $(S,\, +,\, \cdot)$ is a ring, determine whether it is a commutative ring.
(5) If $(S,\, +,\, \cdot)$ is a ring, determine whether it has unity.

Hint. Go by definition. Here we may use the fact (without proof) that, for all complex numbers (which include real numbers), associativity and distribution hold for addition and multiplication.

Problem 1.4. Consider $R = \{\left(\begin{smallmatrix} \alpha & 0 \\ \beta & 0 \end{smallmatrix}\right) \mid \alpha,\, \beta \in \mathbb{R}\}$, which is a ring under the usual matrix addition and matrix multiplication.

(1) Find concrete $a,\, b \in R$ such that $a \neq 0_R$, $b \neq 0_R$ and $ab = 0_R$.
(2) Find concrete $r,\, s,\, t \in R$ such that $rt = st$, $t \neq 0_R$ and $r \neq s$.

Hint. Make sure your matrices are chosen from R (as defined above).

PROBLEMS HINTS SOLUTIONS