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◇ ◇ ◇ ◇ **MATH 4441/6441: MODERN ALGEBRA I** ◇ ◇ ◇ ◇  
**HOMEWORK SETS AND EXAMS**

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HW Set #1, Problems	1
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There are four (4) problems in each homework set. Math 6441 students need to do all 4 problems while Math 4441 students need to do any three (3) problems out the four. If a Math 4441 student submits all 4 problems, then one of the lowest score(s) is dropped. There is a bonus point for Math 4441 students solving all 4 problems correctly/perfectly.

When solving homework problems, make sure that your arguments and computations are rigorous, accurate, and complete. Present your step-by-step work in your solutions/proofs.

There are three (3) PDF files for the homework sets and exams, one with the problems only, one with hints, and one with solutions. Links are available below.

PROBLEMS

HINTS

SOLUTIONS

$$(G, *) \dots H \leq G \dots |G| = [G : H] \cdot |H| \dots a^{|G|} = e \dots \varphi : G \rightarrow G', \varphi(ab) = \varphi(a)\varphi(b) \dots N \trianglelefteq G \dots G/N \dots G/\text{Ker}(\varphi) \cong \text{Im}(\varphi)$$

**Problem 1.1.** Let  $A = \{1, 2, 3, 4\}$ ,  $B = \{2, 3, 5, 6, 7\}$  and  $C = \{3, 6, 7, 8\}$  be sets.

- (1) Compute  $(A \setminus B) \cap C$  and  $A \setminus (B \cap C)$ . Are they equal?
- (2) Compute  $(A \cap B) \cup C$  and  $A \cap (B \cup C)$ . Are they equal?

**Problem 1.2.** Let  $A$ ,  $B$  and  $C$  be sets. Prove  $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$ .

**Problem 1.3.** For each function  $f_i$ , determine whether it is injective but not surjective, surjective but not injective, bijective, or neither injective nor surjective. Explain why.

- (1)  $f_1: \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}$  with  $f_1(x) = x^2$  for all  $x \in \mathbb{R}_{\geq 0}$ , where  $\mathbb{R}_{\geq 0} = \{x \in \mathbb{R} \mid x \geq 0\} = [0, \infty)$ .
- (2)  $f_2: \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$  with  $f_2(x) = x^2$  for all  $x \in \mathbb{R}_{\geq 0}$ .
- (3)  $f_3: \mathbb{R} \rightarrow \mathbb{R}_{\geq 0}$  with  $f_3(x) = x^4$  for all  $x \in \mathbb{R}$ .
- (4)  $f_4: \mathbb{R} \rightarrow \mathbb{R}$  with  $f_4(x) = 10^{x^2}$  for all  $x \in \mathbb{R}$ . (Here  $10^{x^2}$  stands for  $10^{(x^2)}$ , not  $(10^x)^2$ .)

**Problem 1.4.** Let  $A$ ,  $B$  and  $C$  be sets.

- (1) Find a concrete example of  $A$ ,  $B$  and  $C$  such that  $(A \cup B) \cap C \subsetneq A \cup (B \cap C)$ .
- (2) Prove  $(A \cup B) \cap C \subseteq A \cup (B \cap C)$ .

PROBLEMS

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