◇ ◇ ◇ ◇ MATH 4441/6441: MODERN ALGEBRA I ◇ ◇ ◇ ◇
HOMEWORK SETS AND EXAMS

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Contents

There are four (4) problems in each homework set. Math 6441 students need to do all 4 problems while Math 4441 students need to do any three (3) problems out the four. If a Math 4441 student submits all 4 problems, then one of the lowest score(s) is dropped. There is a bonus point for Math 4441 students solving all 4 problems correctly/perfectly.

When solving homework problems, make sure that your arguments and computations are rigorous, accurate, and complete. Present your step-by-step work in your solutions/proofs.

There are three (3) PDF files for the homework sets and exams, one with the problems only, one with hints, and one with solutions. Links are available below.

PROBLEMS HINTS SOLUTIONS

$(G, *) \ldots H \leqslant G \ldots |G| = [G:H] \cdot |H| \ldots a^{|G|} = e \ldots \varphi\colon G \to G',\ \varphi(ab) = \varphi(a)\varphi(b) \ldots N \trianglelefteq G \ldots G/N \ldots G/\operatorname{Ker}(\varphi) \cong \operatorname{Im}(\varphi)$

Problem 1.1. Let $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 5, 6, 7\}$ and $C = \{3, 6, 7, 8\}$ be sets.

(1) Compute $(A \setminus B) \cap C$ and $A \setminus (B \cap C)$. Are they equal?
(2) Compute $(A \cap B) \cup C$ and $A \cap (B \cup C)$. Are they equal?

Hint. All should be straightforward. Determine each set by listing its elements explicitly. (Note that $A \setminus B$ may be also denoted by $A - B$.)

Problem 1.2. Let A, B and C be sets. Prove $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$.

Hint. How to show two sets are equal?

Problem 1.3. For each function f_i, determine whether it is injective but not surjective, surjective but not injective, bijective, or neither injective nor surjective. Explain why.

(1) $f_1 \colon \mathbb{R}_{\geqslant 0} \to \mathbb{R}$ with $f_1(x) = x^2$ for all $x \in \mathbb{R}_{\geqslant 0}$, where $\mathbb{R}_{\geqslant 0} = \{x \in \mathbb{R} \mid x \geqslant 0\} = [0, \infty)$.
(2) $f_2 \colon \mathbb{R}_{\geqslant 0} \to \mathbb{R}_{\geqslant 0}$ with $f_2(x) = x^2$ for all $x \in \mathbb{R}_{\geqslant 0}$.
(3) $f_3 \colon \mathbb{R} \to \mathbb{R}_{\geqslant 0}$ with $f_3(x) = x^4$ for all $x \in \mathbb{R}$.
(4) $f_4 \colon \mathbb{R} \to \mathbb{R}$ with $f_4(x) = 10^{x^2}$ for all $x \in \mathbb{R}$. (Here 10^{x^2} stands for $10^{(x^2)}$, not $(10^x)^2$.)

Hint. Your solution may follow this example: Let $f_5 \colon \mathbb{R} \to \mathbb{R}$ with $f_5(x) = |x|$ for all $x \in \mathbb{R}$. Then f_5 is neither injective nor surjective. It is not injective because $f_5(1) = f_5(-1)$ while $1 \neq -1$. It is not surjective because there is no $x \in \mathbb{R}$ such that $f_5(x) = -2$

Problem 1.4. Let A, B and C be sets.

(1) Find a concrete example of A, B and C such that $(A \cup B) \cap C \subsetneq A \cup (B \cap C)$.
(2) Prove $(A \cup B) \cap C \subseteq A \cup (B \cap C)$.

Hint. (1) For example, you may try letting $A = \{1, 2\}$, $B = \{\dots\}$ and $C = \{\dots\}$. You may even start with $A = \{1\}$. (Note that $X \subsetneq Y$ means $X \subseteq Y$ and $X \neq Y$.)

(2) How to show that a set is a subset of another set? Let $x \in (A \cup B) \cap C$ be an (arbitrary) element. Try to show $x \in A \cup (B \cap C)$.

PROBLEMS HINTS SOLUTIONS