Math 615 – Winter 2005

1:00–2:00pm @ 3088 East Hall, Monday, Wednesday and Friday

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Office Hours: 1:00–3:00pm Tuesday and Thursday at 1851 East Hall. Or by appointment. Or by e-mail.

Textbook: Optional. The following sources, among others, should be helpful:

- M. Hochster's Math 715 lecture notes
  http://www.math.lsa.umich.edu/~hochster/715/715.html
- M. Hochster and C. Huneke, *Tight closure, invariant theory, and the Briançon-Skoda theorem,* Jour. of Amer. Math. Soc. **3** (1990), no. 1, 31–116.
- C. Huneke, *Tight closure and its applications,* CMBS regional Conference Series in Mathematics, no. 88.

Homework: There are about 5–6 homework assignments.

Syllabus: Depending on the interests of the students and the time available, this course will cover topics as follows:

- Tight closure theory. We start by defining the notion of tight closure over rings of characteristic $p > 0$ as well as studying its properties. We will also see how tight closure is used to produce theorems that are hard to prove otherwise (e.g., the Briançon-Skoda theorem in equal characteristic case).
- The theory of Hilbert-Kunz multiplicity. Hilbert-Kunz multiplicity is defined over rings of characteristic $p > 0$ and it is closely related to the tight closure theory. In particular, we will show that an unmixed local ring with Hilbert-Kunz multiplicity equal to 1 (equivalently, sufficiently close to 1) is regular.
- The module structure of R over its subring R^p in the case of characteristic $p > 0$. Topics include rings of finite F-representation type, the theory of F-signature, etc. In particular, a local ring with positive F-signature is strongly F-regular. If the F-signature is equal to 1 (equivalently, sufficiently close to 1), then the ring is regular.
- Integral closures of ideals. We will start from the definition of integral closure and then proceed to study some of its subtle properties. In particular, we aim to present detailed versions of Briançon-Skoda type theorems. Time permitting, we may treat the Lipman-Sathaye Jacobian theorem.

A strong effort will be made to accommodate students with different backgrounds.

Website: http://www.math.lsa.umich.edu/~ywyao/2005W/math615.html