Radiophysics and Quantum Electronics, Vol. 41, No. 12, 1998

SYNCHRONIZED OSCILLATIONS IN A SYSTEM OF TWO COUPLED VAN-DER-POL–DUFFING OSCILLATORS*

D. G. Zakharov, Ya. I. Mol'kov, and M. M. Sushchik

UDC 517.9

The results of analysis of the periodic solutions obtained within the framework of complete and truncated equations for a system of identical Van-der-Pol–Duffing oscillators with nonlinear coupling are compared.

1. The system of coupled Van-der-Pol–Duffing oscillators belongs to the class of HKB models [1], which are used in mathematical simulation of the effects observed in psychophysiological tests [1] and in experiments on the initiation of locomotive movements by nonresonant vibration of muscles [2, 3]. The specificity of the system is determined by the type of nonlinear coupling between interacting oscillators, enabling one to simulate the observed bistability of synchronization modes with the required phase shifts φ_1, φ_2. In the simulation of bistable modes with almost antisymmetric phase shifts $\varphi_1 = +\psi$, $\varphi_2 = -\psi$, analysis can be simplified sometimes by considering only the degenerate case and assuming that the oscillators and their connections are identical:

$$\begin{aligned} \dot{x}_1 &= y_1 - \epsilon\gamma(x_1 - x_2)[(x_1 - x_2)^2 - \alpha], \\ \dot{y}_1 &= \epsilon[y_1(1 - \lambda x_1^2) - \beta x_1^3] - x_1, \\ \dot{x}_2 &= y_2 - \epsilon\gamma(x_2 - x_1)[(x_2 - x_1)^2 - \alpha], \\ \dot{y}_2 &= \epsilon[y_2(1 - \lambda x_2^2) - \beta x_2^3] - x_2. \end{aligned} \tag{1}$$

For small nonlinearity ($\epsilon \ll 1$), a further simplification can be reached by transition to truncated equations. In this case, the solution of system (1) can be represented as

$$\begin{aligned} &x_k = z_k e^{it} + z_k^* e^{-it}, \quad y_k = i\left(z_k e^{it} - z_k^* e^{-it}\right), \quad k = 1, 2, \\ &z_1 = \frac{a}{\sqrt{\lambda}} e^{i\phi_1}, \quad z_2 = \frac{b}{\sqrt{\lambda}} e^{i\phi_2}, \quad \phi = \phi_2 - \phi_1. \end{aligned} \tag{2}$$

The slow amplitudes a, b and the phase difference ϕ are determined by the relationships

$$\begin{aligned} \dot{a} &= a(1 - a^2) - \Gamma(a - b\cos\phi)(a^2 + b^2 - 2ab\cos\phi - A), \\ \dot{b} &= b(1 - b^2) - \Gamma(b - a\cos\phi)(b^2 + a^2 - 2ba\cos\phi - A) \\ \dot{\phi} &= B(b^2 - a^2) - \Gamma\left(\frac{a}{b} + \frac{b}{a}\right)\sin\phi\left(a^2 + b^2 - 2ab\cos\phi - A\right), \end{aligned} \tag{3}$$

where $T = \frac{\epsilon}{2}t$, $B = \frac{3\beta}{\lambda}$, $\Gamma = \frac{3\gamma}{\lambda}$, and $A = \alpha\frac{\lambda}{3}$.

However, it should be borne in mind that in degenerate systems the spontaneous decay of symmetric solutions for initial and truncated equations can be different even in the case of weak nonlinearity, and the solutions will be radically different. Below we consider this difference in the range of parameters for which

*This work was presented at the Summer Workshop "Dynamic Days" (Nizhny Novgorod, June 30–July 2, 1998).

Institute of Applied Physics of the Russian Academy of Sciences, Nizhny Novgorod, Russia. Translated from Izvestiya Vysshikh Uchebnykh Zavedenii, Radiofizika, Vol. 41, No. 12, pp. 1531–1536, December, 1998. Original article submitted October 1, 1998.

0033-8443/98/4112-1037$20.00