A VARIATIONAL MULTIPHASE MODEL FOR SIMULTANEOUS MR IMAGE SEGMENTATION AND BIAS CORRECTION

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ABSTRACT

In this paper, we present a multiphase segmentation model for MR images in the presence of strong intensity inhomogeneity. The problem is formalized as a constraint min-max optimization problem that consists both primal and dual variables. We use the primal dual hybrid gradient (PDHG) algorithm to alternately solve for the optimal solutions. The proposed algorithm is quite efficient in that all the subproblems have closed form solutions. Moreover, the computational complexity is shown to be linear with respect to the size of the image. Numerical experiments on various images demonstrated that our algorithm outperforms recently developed methods in terms of efficiency and accuracy.

***Index Terms*—** Image segmentation, Maximum a posteriori estimation, Optimization, PDHG, Projection algorithms

1. INTRODUCTION

Magnetic resonance (MR) images usually encounter strong intensity inhomogeneity due to limitations in imaging devices or subject-induced susceptibility effect [1]. The intensity inhomogeneity effect is likely to smear object boundaries and thus hampers the process of image segmentation. Therefore, segmentation for MR image is a challenging problem and conventional approaches based on edge detection or intensity classification may not work.

To tackle this issue, various work has appeared for simultaneous segmentation and intensity correction. (e.g. see [1, 2, 3, 4, 5, 6]). Here we review a few models that is closely related to our work. In [5], a variational model was presented for multi-phase tissue clarification of MR images in the presence of intensity inhomogeneity. Their model can be viewed as a local weighted K-means clustering approach and includes membership functions into their energy functional. [7] used the same approach as that in [5] with the only difference being the cost functional was written under the level framework. Recently, another level set approach for simultaneous tissue segmentation and bias correction was proposed in [8]. Different from the work in [5, 7], where the intensities in each cluster were approximated by their means in the L^2 sense, in the model of [8] local intensities of different tissues were assumed to be Gaussian distributed with the means as the centering intensities of each cluster and variances to be optimized. This approach is more general than the one used in [5, 7], which can be viewed as a special case of Gaussian distribution with the same fixed variance for each cluster.

In this paper, we present a multiphase segmentation framework for simultaneous image segmentation and bias correction. We employ the primal-dual formulation and rewrite the minimization problem as a constraint min-max optimization problem that consists both primal and dual variables. We apply the primal dual hybrid gradient (PDHG) algorithm [9] to alternately solve for the optimal solutions. As a result, the numerical algorithm only involves convolutions of small kernel functions, and pointwise projections onto unit ball and canonical simplex. Therefore, all the subproblems have closed form solutions and the proposed algorithm is quite efficient. Numerical experiments on various images demonstrated that our algorithm outperforms recently developed methods in terms of efficiency and accuracy.

2. MODEL FORMULATION

Suppose $I : \Omega \to \mathbb{R}$ is the image to be segmented, where $\Omega \subset \mathbb{R}^2$ is a closed and bounded region that represents the domain of I. Given an image I, the purpose of image segmentation is to partition of its domain Ω into multiple (say M) regions, such that each region delineates an object distinct from others. Namely, we need to solve for a set of regions $\{\Omega_i\}_{i=1}^M$ such that $\Omega = \cup_{i=1}^M \Omega_i$, $\{\Omega_i\}_{i=1}^M$ are disjoint. In other words, we need to solve for membership functions of the form $u = (u_1, \cdots, u_M)^T : \Omega \to \{0, 1\}^M$ subject to $\sum_i u_i(x) = 1$ for each point $x \in \Omega$, where u_i is the characteristic function of region Ω_i.

As stated in the introduction, the image I can be corrupted by some unknown intensity bias field $b : \Omega \to \mathbb{R}$ and noise. In this paper, we further assume the intensities of original clean image are constant c_i at each region Ω_i, and the noise is normally distributed and independent of those at other locations, which can be mathematically modeled by

$$I(x) = b(x)c_i + n_i(x), \tag{1}$$

where $n_i(x)$ is normally distributed with mean 0 and variance σ_i^2 unknown.

Denote $c = (c_1, \dots, c_M)^T$ and $\sigma = (\sigma_1, \dots, \sigma_M)^T$. Our purpose is to find the posterior probability distribution $p(\{u, b, c, \sigma^2\}|I)$ of the segmentation results $\{u, b, c, \sigma\}$ given image I. By the Bayes' rule, we have

$$p(\{u, b, c, \sigma\}|I) \quad \propto \quad p(I|\{u, b, c, \sigma\})p(\{u, b, c, \sigma\}), \tag{2}$$

where $p(\{u, b, c, \sigma\})$ is the prior information imposed to the segmentation $\{u, b, c, \sigma\}$, and $p(I|\{u, b, c, \sigma\})$ is the joint distribution of pixel intensities in I given the segmentation $\{u, b, c, \sigma\}$.

According to (1), it is easy to see that $I(x)$ is normally distributed as $N(b(x)c_i, \sigma_i^2)$ if $u_i(x) = 1$ given the segmentation $\{u, b, c, \sigma\}$. However, the observed intensity $I(x)$ is merely one realization and it is usually not reliable to recover u, b, c and σ_i simultaneously. To overcome this difficulty, we assume a multiplicative density structure of $I(x)$ as follows,

$$p(I(x)|\{u, b, c, \sigma\}) \propto \prod_{y \in W_x^\rho} \left(p(I(y)|\{u, b, c, \sigma\}\right)^{1/|W_x^\rho|}, \tag{3}$$

where $W_x^\rho = \{y \in \Omega : |y - x| \leq \rho\}$ is the set of points centered at x with a prescribed radius ρ and $|W_x^\rho|$ refers to the number of points in W_x^ρ. On the right side of (3), we assume that $I(y)$ closely follows the model (1).

Note that the intensity bias field varies gradually across the image domain, we can approximate $b(y)$ by $b(x)$ and obtain that $I(y) \sim N(b(x)c_i, \sigma_i^2)$ for $y \in W_x^\rho \cap \Omega_i$. As a result, the joint distribution $p(I|\{u, b, c, \sigma\})$ can be written as

$$p(I|\{u, b, c, \sigma\}) = \prod_{x \in \Omega} \prod_{y \in W_x^\rho} p(I(y)|\{u, b, c, \sigma\})^{1/|W_x^\rho|}, \tag{4}$$

where $p(I(y)|\{u, b, c, \sigma\})$ is of Gaussian-type $N(b(x)c_i, \sigma_i^2)$.

On the other hand, we set the priors of b, c and σ to be (non-informative) uniform distributions, and the prior of u according to the descriptive length of the boundaries $\partial\Omega_i$ to exponential distribution with parameter λ, which implicitly penalizes irregular and zigzag partition curves. Moreover, terms in $\{u, b, c, \sigma\}$ are assumed to be independent. Therefore, the prior $p(\{u, b, c, \sigma\})$ can be simplified to

$$p(\{u, b, c, \sigma\}) \propto \prod_{i=1}^{M} \exp(-\lambda|\partial\Omega_i|). \tag{5}$$

Based on (4) and (5), the MAP of (2) is equivalent to the following minimization problem,

$$\min_{u,b,c,\sigma} \left\{ \lambda \sum_{i=1}^{M} |\partial\Omega_i| + L(\{u, b, c, \sigma\}) \right\}, \tag{6}$$

where $L(\{u, b, c, \sigma\})$ is the negative log-likelihood function

$$\begin{aligned} L(\{u, b, c, \sigma\}) &= -\log p(I|\{u, b, c, \sigma\}) \\ &= \frac{1}{|W_x^\rho|} \sum_{i=1}^{M} \int_\Omega \int_\Omega u_i(y) l_i(y; x) \mathrm{d}y \mathrm{d}x \\ &\approx \frac{1}{|W_x^\rho|} \sum_{i=1}^{M} \int_\Omega \int_{W_x^\rho} \mathcal{K}_s(y - x) u_i(y) l_i(y; x) \mathrm{d}y \mathrm{d}x, \end{aligned} \tag{7}$$

where $l_i(y; x)$ is defined by

$$l_i(y; x) := \frac{|I(y) - b(x)c_i|^2}{2\sigma_i^2} + \frac{1}{2}\log(2\pi\sigma_i^2), \tag{8}$$

and $\mathcal{K}_s$ a (truncated) Gaussian kernel function defined by

$$\mathcal{K}_s(z) = \begin{cases} k \exp\left(-|z|^2/2s^2\right), & \text{if } |z| \leq s \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

for some $s > 0$. In (9), k is a normalizing constant that makes $\int_{|z| \leq \rho} \mathcal{K}_s(z) \mathrm{d}z = 1$.

The first term in (6) can be expressed by the total variation,

$$|\partial\Omega_i| = TV(u_i) =: \sup_{p \in Y} \left\{ -\int_\Omega u_i \mathrm{div} p \mathrm{d}x \right\}, \tag{10}$$

where the admissible set Y is

$$Y := \{p \in \mathcal{C}_0^\infty(\Omega; \mathbb{R}^d) : |p(x)| \leq 1, \forall\, x \in \Omega\}. \tag{11}$$

Plug (10) and (7) into (6), we obtain the multiphase segmentation model as follows,

$$\min_{u,b,c,\sigma} \sum_{i=1}^{M} \left\{ \lambda TV(u_i) + \int_\Omega u_i(x) h_i(x) \mathrm{d}x \right\} \tag{12}$$

subject to the constraint that only one component in $u(x) = (u_1(x), \dots, u_M(x))^T$ is one and the rests are zeros. In (12), the function h_i is defined as

$$h_i(x) = \frac{1}{|W_x^\rho|} \int_\Omega \mathcal{K}_s(y - x) l_i(x; y) \mathrm{d}y. \tag{13}$$

3. NUMERICAL ALGORITHM

In this section, we develop an efficient iterative algorithm to solve the constrained minimization problem (12). We first relax the constraint on the function $u = (u_1, \cdots, u_M)^T$ in (12) to X defined by

$$X := \{u : \Omega \to [0, 1]^M | u(x) \in \Delta^M, \forall\, x \in \Omega\} \tag{14}$$

and the canonical simplex Δ^M is defined by

$$\Delta^M = \{(z_1, \cdots, z_M)^T \in \mathbb{R}_+^M : z_1 + \cdots + z_M = 1\}. \tag{15}$$

One can easily see that the original constraint requires that $u(x)$ to be one of the vertexes of Δ^M and this relaxation extends u to a continuous domain. However, this is already good enough as we can utilize the primal-dual gradient projections to construct a fast iterative algorithm.

We apply the alternating minimization scheme to solve problem (12). For fixed u, the variables b, c, and σ only appear in the second term of (12). Hence, $b(x)$, c_i, and σ_i^2 can be updated by calculating their first variations.

$$\begin{aligned} b &= \frac{\sum_{i=1}^M (c_i/\sigma_i^2)[\mathcal{K}_s * (u_i I)]}{\sum_{i=1}^M (c_i^2/\sigma_i^2)[\mathcal{K}_s * u_i]}, \\ c_i &= \frac{\int_\Omega [\mathcal{K}_s * (u_i b I)](x)\mathrm{d}x}{\int_\Omega [\mathcal{K}_s * (u_i b^2)](x)\mathrm{d}x}, \\ \sigma_i^2 &= \frac{\int_\Omega \left([\mathcal{K}_s * (u_i I^2)] - 2c_i b[\mathcal{K}_s * I] + c_i b^2\right)\mathrm{d}x}{\int_\Omega [\mathcal{K}_s * u_i]\mathrm{d}x}. \end{aligned} \tag{16}$$

Now let us turn to the u subproblem. For fixed b, c and σ, the minimization can be written as

$$\min_{u\in X} \sum_{i=1}^M \left\{ \lambda TV(u_i) + \int_\Omega u_i(x) h_i(x)\mathrm{d}x \right\}. \tag{17}$$

This is a constrained nonsmooth optimization problem due to the constraint on $u(x) \in \Delta^M$ for each $x \in \Omega$ and the non-differentiable TV term in the objective function. So we need to seek for an effective way to tackle these two issues.

For each u_i, we introduce the dual variable $p_i \in Y$ according to the definition in (10), and reformulate the minimization problem (17) as a min-max problem

$$\min_{u\in X} \max_{p_i \in Y} \sum_{i=1}^M \left\{ -\lambda \int_\Omega u_i \mathrm{div} p_i \mathrm{d}x + \int_\Omega u_i h_i \mathrm{d}x \right\}. \tag{18}$$

where X is defined in (14) and Y is the defined in (11).

In the discrete setting where the image to be segmented consists of N pixels, we can vectorize each u_i into a column vector in $\mathbb{R}^N$, then its dual variable p_i is a matrix in $\mathbb{R}^{N\times d}$, where d is the dimension of the image (e.g. 2 or 3). Hence, the optimization problem (18) can be written as

$$\min_{u\in X} \max_{p_i\in Y} F(u,p) := \sum_{i=1}^M \langle u_i, \lambda D^T p_i + h_i \rangle. \tag{19}$$

where $D : \mathbb{R}^N \to \mathbb{R}^{N\times d}$ is the discretized gradient operator, the superscript ^{T} is the conjugate operator, and $\langle \cdot, \cdot \rangle$ represents the regular inner product in $\mathbb{R}^N$.

Both of X and Y are closed and convex sets and thus a solution to (19) can be obtained by alternately solving for the primal variable u and dual variable p

$$\begin{aligned} u^{k+1} &= \Pi_X(u_i^k - \delta_k \nabla_{u_i} F(u^k, p^k)), \\ p_i^{k+1} &= \Pi_Y(p_i^k + \tau_k \nabla_{p_i} F(u^{k+1}, p^k)), \end{aligned} \tag{20}$$

where δ_k and τ_k act as the step sizes of the primal and dual variables u and p in the k-th iteration, respectively, and $\|\cdot\|_2$ is the regular Euclidean norm of vectors. We note that the solution to each problem has closed form as

$$\begin{aligned} u_i^{k+1} &= \Pi_X(u_i^k - \delta_k(\lambda D^T p_i^{k+1} + h_i/\lambda)), \\ p_i^{k+1} &= \Pi_Y\left(p_i^k + \tau_k D u_i^k\right), i = 1, \cdots, M, \end{aligned} \tag{21}$$

where $\Pi_X : \mathbb{R}^{N\times M} \to X$ and $\Pi_Y : \mathbb{R}^{N\times d} \to Y$ are projection operators onto the sets X and Y, respectively. More precisely, Π_X maps each row of its argument, say $z \in \mathbb{R}^M$, to the simplex using algorithm proposed in [10], and Π_Y projects each row of its argument to the unit ball B^d. These projections are applied to each of the N pixels and hence the computations in both of Π_X and Π_Y can be carried out in parallel.

4. EXPERIMENTAL RESULTS

In this section, we test the proposed model on a series of real MR images and compare it with some recently developed methods.

In Fig.1, we compare the proposed model with SVMLS [8]. The original test image (a) suffers strong intensity inhomogeneity as indicated in its histogram (d). We use the same initial condition (g) for both methods. The intensity inhomogeneity has been suppressed a lot in the bias corrected images (b) and (c) in that the corresponding histograms (e) and (f) have three sharp peaks. Therefore, both models are effective for bias correction. The segmentation result of the proposed model (i) is much more accurate than that of the SVMLS (h) as indicated in the subregion pointed out by the red arrow.

Fig.2 shows the comparison with the CLIC (coherent local intensity clustering) model proposed in [5]. Strictly speaking, the CLIC model is just the fidelity term of our model (7)(8) with $\sigma_i \equiv 1$. Another thing worth noting is that the CLIC model has no regularization regarding the membership function u and they manually set u to be binary during the iterations. The test images are generated by adding Gaussian white noise (with variance 0.001 and 0.01 respectively) to the original clean MR image. As shown in Fig.2, both of the two models work well for the test image with low noise. However, the CLIC model fails when we increase the noise level.

5. CONCLUDING REMARKS

In this paper, we present a novel multiphase segmentation framework for images with severe intensity inhomogeneity and noise. The constrained minimization problem is solved by employing the primal-dual hybrid gradient method. It is shown that all the variables have closed form solutions and the resulting algorithm could be paralleled. Numerical results on various images show that our method is more efficient and accurate in comparison with recently proposed algorithms.

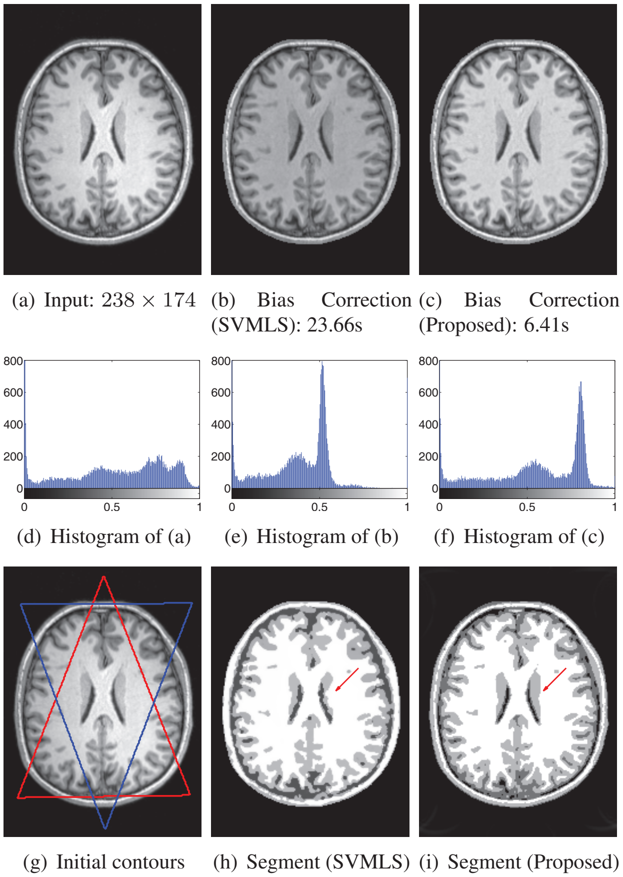

Fig. 1. Comparison with SVMLS [8] on an MR brain image.

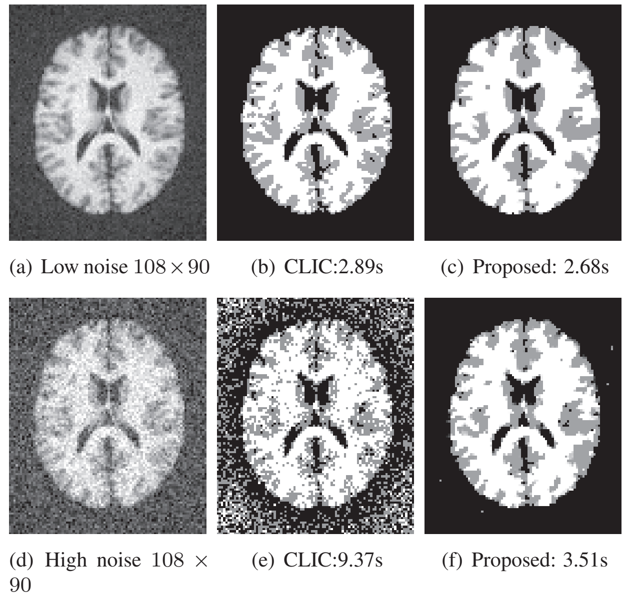

Fig. 2. Comparison with CLIC on different level of noise.

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