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LP Shilnikov 1934–2011: A Tribute From 'The West'

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This article presents a tribute to L. P. Shilnikov from a fan, a scientist from the next generation who owes much of his career to the results pioneered by this truly great man. The discourse is based on a mixture of personal recollections and conversations with a number of key individuals.

Keywords: Homoclinic bifurcation; chaos; Soviet science.

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It is hard to believe that over two years have passed since, on 26th Dec 2011, the world lost one of the true pioneers of nonlinear dynamical systems theory; Leonid Pavlovich Shilnikov, known simply to his friends as LP (pronounced more like "*elpér*" in Russian). He died of cancer, at home in Nizhny Novgorod surrounded by his family, nine days after passing his 77th birthday.

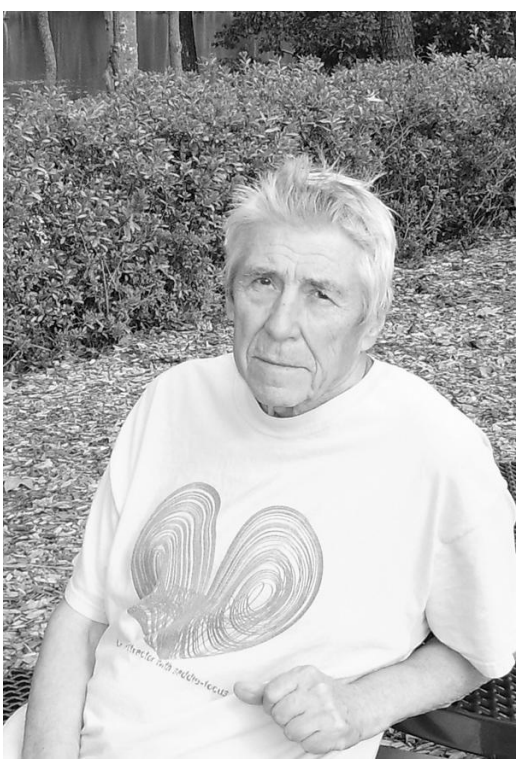

Fig. 1. L. P. Shilnikov in 2010.

This article is not intended as a formal obituary, although an earlier version of it appeared shortly after LP's death in the DSWeb magazine [Champneys, 2011]. A far more detailed biographical sketch, including a range of photographs of the man and more specifics on his mathematical achievements, can be found in the various contributions to the commemorative volume [Afraimovich *et al.*, 2010] that appeared to mark his 75th birthday. Nor is this article intended as a personal tribute as such. It would hardly be fitting because I believe I only met the man once, when I was a graduate student, and then I was far too shy to attempt to speak to him in person. Rather, this is intended as an affectionate view from a fan, someone who in some respects owes his career (at a University which in the UK is known as being in the 'West Country' — hence this article's title) to the mathematics that LP pioneered. As will become apparent, I also feel that history was in many ways unkind to LP during his lifetime and his reputation deserves to be more widely recognized. But that is of course a personal view, and it is only the eyes of history that can truly judge.

Most of what follows arose from a series of conversations I had with LP's son Andrey, starting

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I believe in 2010, as part of a book project that I am (very slowly) writing which aims to popularize the concept of homoclinc orbits and localized phenomena in a variety of scientific contexts. I was honored that LP himself saw my draft book chapter and provided comments — correcting certain falsehoods and re-emphasizing other points. Further facts were supplied by Andrey and Dmitry Turaev shortly after LP's death, for inclusion in the DSWeb article.

When I took a first year graduate course in nonlinear dynamics in the late 1980s, I learnt of Lorenz attractors and Smale horseshoes. Both were discovered in the early 1960s, and were pivotal to our current geometric understanding of dynamical systems that feature chaotic dynamics. It seems that when "chaos theory" revolution began in the West in the 1970s, it was not clear how the horseshoe and the Lorenz attractor relate to one another. But, unknown to us in the West, at the same time as Ed Lorenz and Steve Smale made their breakthroughs, a young mathematician in Gorky had made a remarkable discovery which provides just such a link. Not only that, the unfolding of the dynamics close to a saddle-focus homoclinic bifurcation that now universally bears the name Shilnikov, provides a universal mechanism for the onset of chaos.

It seems hard now to remember how scientific results spread (or did not) in the days before the Internet. Things were also exasperated by the Cold War. So, whereas Lorenz and Smale received almost universal acclaim, L. P. Shilnikov worked in a city deep within the USSR that was closed to foreigners. In those days, some top Russian scientists were allowed foreign travel by the Soviet authorities. But L. P. Shilnikov was not one of them. In fact, it took two decades before his work gained the recognition it deserved, especially in the West. As a result, it would seem that popular accounts on the historical development of "chaos theory" often overlooked LP's pivotal contributions.

LP was a member of the now famous Andronov school of Russian mathematicians and physicists who applied and extended many of Poincaré and Lyapunov's topological methods for analyzing dynamics. These people worked mostly in the Gorky Institute of Physical-Technical Researches (GIFTI) which was founded by the Moscow-trained physicist Aleksandr A. Andronov. In 1931, accompanied by his mathematician wife Evgeniya Leontovich, Andronov moved to the city of Nizhny Novgorod. Literally translated as "lower new town" and known to locals merely as Nizhny, it is an industrial city, the third largest in Russia, situated some 300 miles East of Moscow. The year after the Andronovs arrived, under Stalin's orders, Nizhny Novgorod was renamed Gorky. And Gorky it remained until the fall of the Soviet era, when, in 1990, like Leningrad, the city reverted to its original name.

The Andronov school developed a comprehensive theory of nonlinear oscillations for systems with two state variables. Their results were collected in 1937 into the now classic monograph that Andronov co-authored with Aleksandr Vitt and Semën Khaikin (Vitt's name never appeared in the first edition of this work, he was a victim of Stalin's purges and died in a Siberian labor camp in 1938. It was not until 1966, that the work was fully translated into English, by which time Vitt's name was finally given its rightful place among the authors).

By the time of Andronov's premature death in 1952, his School had grown into a large and complex organization. It had expanded in remit too; to cover many areas of physics, mathematics and engineering in which nonlinear oscillations were important. Andronov's widow, by then known as Evgeniya Leontovich–Andronova, petitioned the Russian Academy of Sciences to form a dedicated theory core, the "Institute of Mathematics and Cybernetics". Her wish was granted and she became head of the Department of Differential Equations, with the explicit goal to continue the tradition of classifying different kinds of nonlinear oscillations using qualitative methods. In particular, she completed and published work with her late husband on classifying all cases of dynamics near homoclinic loops of systems of differential equations on a plane. The focus of her department's activity then switched to studying how these ideas extended to higher-dimensional phase space.

Higher-dimensional generalizations of the Andronov–Leontovich theory of homoclinic loops became the subject of Shilnikov's PhD research project. He switched to this topic from his earlier works (with Yuri Neimark) on perturbation methods and piecewise-linear systems. These were important themes in automatic control theory, which flourished in the late 50s, but he found the field both boring and too crowded. Homoclinic bifurcations were a different matter. Andronov–Leontovich theory used topology of the plane

Fig. 2. L. P. Shilnikov in the 1960s.

heavily, and extending the theory to make it free from Poincaré–Bendixson-type arguments was a real challenge at the time. The first results were more or less the same as in 2D; the bifurcation of single, regular, isolated periodic orbits. Then, in January 1963, shortly after defending his PhD, the 28-year-old L. P. Shilnikov made his key discovery.

He looked at homoclinic trajectories to a saddle-focus equilibrium in three dimensions. Such points contain complex eigenvalues in their linearization and, if these have weaker real parts than the opposing real eigenvalue, Shilnikov found that the corresponding homoclinic loop implies chaotic dynamics. Specifically, he could prove that the chaotic dynamics are governed by a Smale horseshoe. In fact, not one Smale horseshoe; but an infinite number of different Smale horseshoes (more precisely, a Bernoulli shift on infinitely many symbols). This appears to be the first mathematically rigorous method for generating chaos in dynamical systems that do not have an underlying periodic forcing like the van der Pol oscillator. Over the years, Shilnikov's mechanism of chaos has proven to be one of the most robust and frequently occurring mechanisms chosen by nature.

The discovery came as a shock. Steve Smale's early ideas on the horseshoe had already reached Gorky after Leontovich–Andronova attended his talk at a conference in Kiev in 1961 (she remarked in passing that Smale reminded her of Huckleberry Finn). However nobody could expect that such dynamics, which contains infinitely many different periodic motions, could be a necessary consequence of a generic homoclinic bifurcation. Leontovich–Andronova recounted to Shilnikov very much later her first reaction "I immediately wanted to say that this simply cannot be!"

Shilnikov's remarkable result was presented in rather short form in the Doklady Akedemii Nauk SSSR in 1965, with the full results and complete proof appearing in 1970. Within these few years Shilnikov produced a string of papers which included the study of the dynamics that is implied by Poincaré homoclinic tangles, the extension of his saddle-focus result to arbitrary n-dimensional systems, and many other kinds of homoclinic bifurcations. More and more results came out. Leontovich–Andronova referred to him as "a Mozart" (quoting Pushkin "you, Mozart, are a god, and you don't know it"), such was his capacity to discover more and more fascinating mechanisms that generate complex dynamics. He began to attract the first of a succession of talented PhD students; Nikolai Gavrilov, Valentin Afraimovich, Lev Lerman, Vyacheslav Grines, Leonid Belyakov, Vadim Bykov, Albert Morozov, Valery Lukyanov, Sergey Gonchenko, Mikhail Malkin, Nikolai Roschin, Dmitry Turaev, Ilya Ovsyannikov, Valery Biragov, Yuri Komlev, Igor Belykh, Mikhail Shashkov, Oleg Sten'kin, Vladimir Gonchenko (Sergey's son) and LP's own son Andrey.

Science was spoken openly and democratically in the Shilnikov seminars in Gorky in the 1970s and 80s. Late in the evenings, fellow scientists would come to LP's apartment and talk mathematics into the wee hours, chain smoking and tea drinking while he was pacing up and down in his kitchen. One day, in 1976, the Moscow mathematician Yakov Sinai visited the Shilnikov group as part of a PhD defence committee. After the formal business, as they walked back to LP's home, Sinai told him about the Lorenz attractor. LP was fascinated. He saw straight away that what Lorenz had observed also fit into his theory of homoclinic bifurcations. He set his student Bykov, who had experience programming computers, on the task of computing the homoclinic curves in the system.

At the same time with Bykov and his former student Afraimovich, he showed theoretically what

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Fig. 3. L. P. Shilnikov in the 1980s.

lay behind the shape of the butterfly strange attractor. By a bizarre twist of the Soviet era scientific life, their paper had to lay for five years with the publisher. Eventually, in the early 1980s, this work received wide dissemination within the Soviet Union when LP included his new-found understanding of the Lorenz attractor as a substantial Appendix to the Russian translation of a book on applications of the Hopf bifurcation by Marsden and McCracken.

While LP's fame within Russian academia spread, he was barely tolerated at his home University. His success at a young age had broken the mould. His great breakthrough was arrived at alone, and he did not add the name of his Master's supervisor Neimark onto his key papers, as he had done with his earliest work. Trumped up personal allegations against LP surfaced. He was being watched. In 1970, he applied to receive a Doctor of Science (DSc.) degree, a high honor and a necessary requirement to obtain the status and salary equivalent to "full professor" in the US system. There was a four-year delay before his application was reviewed. He openly mentioned mistakes in several of Neimark's papers, and Neimark took serious offence. The jury was split, and he was turned down. After this setback, he never tried again to obtain a DSc. Nor was he ever elected to be a fellow of the Russian Academy of Sciences (despite winning their prestigious Lyapunov award in 1998). It would seem that LP's open scientific approach, interested only in the truth and never playing political games, was to his own detriment.

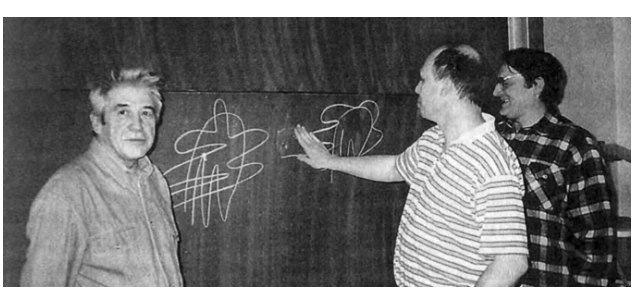

Fig. 4. LP at the blackboard in Berlin in 2004 with D. Turaev and S. Gonchenko.

Over the years, news of Shilnikov's work began to filter out slowly in the West, following translation of the Doklady Akedemii into English. As his fame spread, LP would receive invitations to give keynote talks at international conferences. But he was never allowed to go. Typically, the invitation letters would arrive already opened and with the date of the conference having already passed.

His work gained further impact through the results of two PhD students, working independently in the early 1980s. Paul Glendinning was studying with Colin Sparrow at Cambridge. Simultaneously, Pierre Gaspard was studying under Gregoire Nicolis at the Free University of Brussels. Both were given the task of revisiting and understanding the translated Russian papers of one L. P. Shilnikov. With the aid of computers and modern graphics, both were able to depict the geometry of periodic orbits close to a saddle-focus homoclinic orbit as discovered by LP in the 1960s. When Glendinning and Gaspard became aware of each other's work, it was arranged that their key findings would be simultaneously submitted to the same journal. Indeed, eventually the papers appeared back to back in the seemingly unlikely *Journal of Statistical Physics* (remember, in 1984 there were yet to be any specialist journals of nonlinear dynamics and chaos). At around the same time the Frenchman Charles Tresser published related results in the *Annals of the Henri Poincaré Institute*. In the next few years, this particular Shilnikov mechanism was found to explain the source of complex dynamics in a wide variety of different physical systems; in chemistry, in fluid mechanics, in laser instabilities, and in optical pulse propagation.

Everything changed in Gorky after Glasnost, Perestroika, and the fall of the iron curtain. In 1990,

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LP was finally allowed to visit the West. He was invited by Neal Abraham in the US to attend a conference on nonlinear optics. Abraham took the trouble to fly to Moscow to fetch LP. Then, in 1991, a conference was organized in Brussels in Shilnikov's honor. He was accompanied by his former students Lev Lerman, Vadim Bykov and by his son Andrey.

There was a real sense of "East meets West". Perhaps what was most interesting though — and I was there, this being the one occasion I met LP — was the eclectic range of applications that were presented in which homoclinic bifurcations to a saddle-focus provided the key to what was observed, both in mathematical models and in experiments. There were talks on chemistry, fluid mechanics, neuroscience, combustion, lasers, and even astrophysics. The proceedings of the meeting [Gaspard *et al.*, 1993] serve as a detailed exposition of just how important the Shilnikov mechanism had proved to be in understanding chaotic dynamics and pattern formation across science. LP was hailed as a hero and, in the evenings, much vodka was drunk in his honor.

LP though, while greatly touched, expressed surprise that most of the talks were on applications, and that there had been seemingly little new mathematical development of homoclinic bifurcation theory in the West. While this was partially true, and indeed many of the latest theoretical developments had been due to Shilnikov's colleagues and students, theoretical activity was taking place elsewhere, not represented at the conference. For example, Xiao-Biao Lin in the US and Bjorn Sandstede in Germany were developing complementary methods for analyzing Shilnikov-related phenomena in other kinds of evolution models such as functional and partial differential equations.

Perhaps LP's comments stemmed from an incident which Paul Glendinning recounted to me that occurred during one late-night vodka drinking session. During the evening Paul, Colin Sparrow and a few other Western colleagues engaged in scientific discussion with their Russian counterparts. Given the lack of a common language, they turned to drawing pictures interspersed with occasional mathematical symbols to describe different cases of homoclinic bifurcations that they were aware of. Both East and West rather proudly wrote next to each of the diagrams the year in which that particular case had first been analyzed by scientists within their respective spheres of influence. To the chagrin of the Western Europeans, it would seem that time after time, the Russians had got there first; sometimes, many years in advance of the Western rediscovery. Glendinning could recall only one solitary case for which, arguably, Western scholars had scooped those of Shilnikov's school.

Fig. 5. LP at work in Andrey's home in 2010.

In the last 20 years of his life, L. P. Shilnikov received many plaudits for his work. He wrote more than 200 scientific publications. Many of the fruits of his work are made accessible in the two-volume book [Shilnikov *et al.*, 1998, 2001] published with Andrey, Dimitry Turaev and Leon Chua in 1998 and 2001.

LP lived out his final years in Nizhny, with his wife Lyudmila, close to their daughter and her grown-up family. His son Andrey, by now a tenured mathematical neuroscientist at Georgia State University, was a frequent visitor. LP continued to publish original research. He traveled freely, and further conferences were held in his honor to mark both his 70th and 75th birthdays. Following a few health scares, he gave up smoking and until his final battle with cancer, lived a happy and fulfilled existence, with fishing being his second passion after mathematics. One of LP's former students Valery Biragov, who had become a priest and changed his name to Hegumen Vassian, gave communion to LP the day before he died and also conducted his funeral service.

I shall always regret that I did not get to know L. P. Shilnikov personally, given the influence his

Fig. 6. LP with his wife Lyudmila in 2010.

work had on my career. I also regret that I never got to visit Nizhny Novgorod during his lifetime. It was a delight though to be present at the meeting held in LP's honor in the very hot beginning of July 2013. There will doubtless be other contributions to this volume that will deal with the science discussed at that conference. But I would rather end on a personal note. Despite not feeling particularly well during the meeting, for reasons that I do not need to go into here, there was part of the experience that felt to me like a pilgrimage; it was as if I was walking on hallowed turf. And I shall always treasure the memory of drinking a toast there to LP's memory, with so many of his former colleagues and students.

Acknowledgments

I should like to thank Andrey Shilnikov for providing me with so much detailed information, and indeed the photographs from the Shilnikov family album that have been included here. I am also honored that L. P. Shilnikov was able to provide comments and corrections to an earlier draft that is destined for a book that I am slowly writing for Oxford University Press (I also thank OUP for their infinite patience). I also acknowledge helpful comments from Paul Glendinning, Colin Sparrow, Lev Lerman, Gregoire Nicolis and Dima Turaev.

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