Supplementary material for "Spectral Network Principle for Frequency Synchronization in Repulsive Laser Networks"

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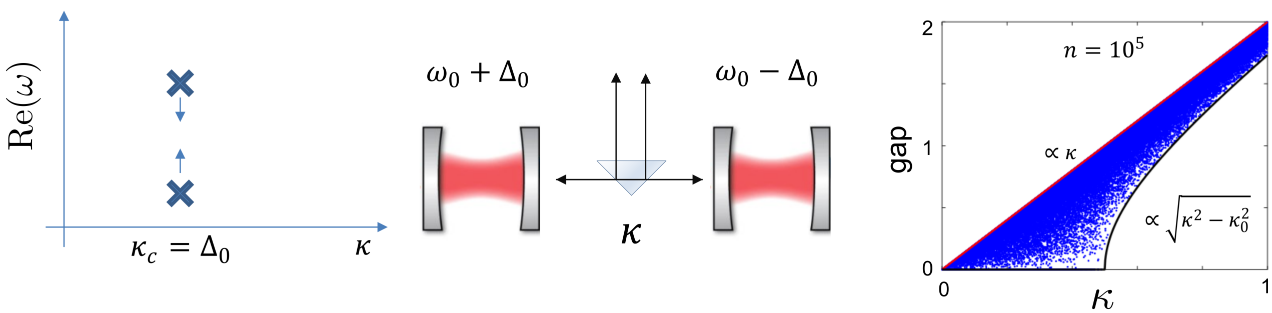
TWO-OSCILLATOR NETWORK WITH RANDOM DETUNING

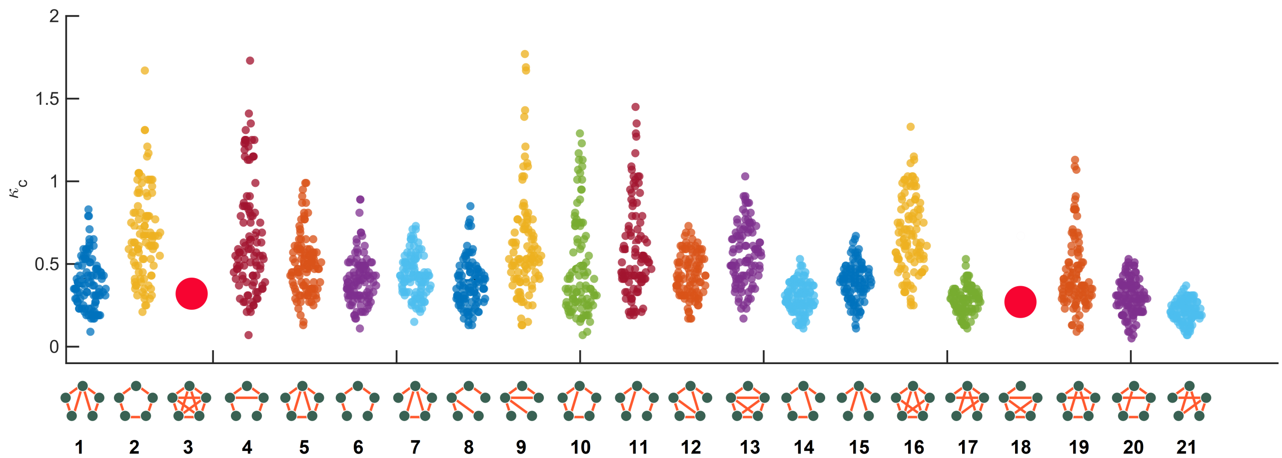
This section provides an additional illustration of the spectral network principle (7) for detuned oscillators. Supplementary Figure 1 shows how the spectral gap γ_M depends on the random detuning and the coupling strength κ.



Supplementary Figure 1. (Left): Synchronization threshold $\kappa_c = \kappa_0 \equiv \Delta_0$ of two detuned laser oscillators with frequencies $\omega_1 = \omega_0 - \Delta$ and $\omega_2 = \omega_0 + \Delta$. Decreasing the frequency mismatch lowers the synchronization threshold. (Middle): An exemplary implementation of the dissipative coupling for a pair of laser oscillators (microring resonators). The coupling is mediated through a scattering element which causes radiation leakage. (Right): The spectral gap γ_M as a function of coupling strength κ in the two-oscillator network with random frequency detunning $\pm\Delta$ for $-\Delta_0 \leq \Delta \leq \Delta_0$ for $n = 10^5$ trials. The gap is located between $\Re[\sqrt{\kappa^2 - \kappa_0^2}] \leq \gamma \leq \kappa$ and visualizes the phase transition. Other parameters $\omega_0 = 1$ and $g_0 = 0.02$.

FREQUENCY SYNCHRONIZATION IN THE KURAMOTO-TYPE MODEL OBTAINED FROM THE LASER MODEL (EQ. (1))

Supplementary Figure 2 is similar to Fig. 4 but calculated for the Kuramoto-type phase model.



Supplementary Figure 2. Synchronization threshold κ_c for all 21 possible connected networks of five detuned Kuramoto-type phase model oscillators obtained from the phase equation (3) by setting $A_n = A_m = 1$. The simulations, notations, and parameters are identical to those of Fig. 4 in the main paper. Note that the phase model networks generally have similar synchronization properties determined by the spectral gap as their full model counterparts of Fig. 4, with one major exception of the locally coupled ring (network 3), which is synchronizable in the case of phase oscillators (this figure) and unsynchronizable in the case of full laser oscillator model (1) (Fig 4).