Atlanta Lecture Series in Graph Theory and Combinatorics XXIX

Georgia State University

November 2 - 3, 2024

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Proem

The Atlanta Lecture Series (ALS) in Combinatorics and Graph Theory is a cornerstone event for the combinatorics community in the southeastern United States and beyond. This prestigious series fosters collaboration and knowledge exchange among researchers and institutions in the region.

Originally, the series had three mini-conferences annually, rotating among Emory University, Georgia Institute of Technology, and Georgia State University. Later the format was adjusted to two mini-conferences per year, continuing the rotation among the three institutions. Supported by the National Science Foundation, this conference series highlights cutting-edge research in areas such as structural graph theory, extremal graph theory, random graphs, hypergraphs, and related fields.

Each mini-conference in the series features a principal speaker, along with several leading researchers and emerging talents in combinatorics and graph theory, offering a platform to showcase innovative ideas and foster the next generation of scholars.

Useful Information

Venue:

Room 124, Petit Science Center, 145 Piedmont Ave SE, Atlanta, GA 30303

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Session I

Schedule

Saturday Morning		
Time	**Speaker**	**Title**
8:00-8:25	Shunzhe Zhang	Chords of longest cycles passing through a specified small set
8:30-8:55	Arthur Tanyel	A degree sequence condition for Hamiltonicity in tough graphs
9:00-9:25	Yixuan Huang	The Chvátal-Erdős condition and Hamiltonicity of Cartesian products
9:30-9:55	Brittian Qualls	Unavoidable immersions in finite and infinite graphs
10:00-10:25	Yuying Ma	A note on bipartite graphs with the double Hall property
10:30-10:55	Ioannis Eleftheriadis	First-order model checking on monadically stable graph classes
11:00-11:25	Sean Longbrake	Supersaturation, enumeration and randomness
11:30-11:55	Yantao Tang	The number of independent transversals in multipartite graphs

Saturday
8:00-8:25

Chords of longest cycles passing through a specified small set

Shunzhe Zhang

University of Mississippi

A long-standing conjecture of Thomassen says that every longest cycle of a 3-connected graph has a chord. Thomassen (2018) proved that if G is 2-connected and cubic, then any longest cycle must have a chord. He also showed that if G is a 3-connected graph with minimum degree at least 4, then some of the longest cycles in G must have a chord. Zhang (1987) proved that if G is a 3-connected simple planar graph which is 3-regular or has minimum degree at least 4, then every longest cycle of G must have a chord. Recently, Li and Liu showed that if G is a 2-connected cubic graph and x, y are two distinct vertices of G, then every longest (x, y)-path of G contains at least one internal vertex whose neighbors are all in the path. In this paper, we study chords of longest cycles passing through a specified small set and generalize Thomassen's and Zhang's above results. We also extend the above-mentioned result of Li and Liu for 2-connected cubic graphs. This is joint work with Haidong Wu.

Saturday
8:30-8:55

A degree sequence condition for Hamiltonicity in tough graphs

Arthur Tanyel

Auburn University

Generalizing both Dirac's condition and Ore's condition for Hamilton cycles, Chvátal in 1972 established a degree sequence condition for the existence of a Hamilton cycle in a graph. Hoàng in 1995 generalized Chvátal's degree sequence condition for 1-tough graphs and conjectured a t-tough analogue for any positive integer $t \geq 1$. Hoàng in the same paper verified his conjecture for $t \leq 3$ and recently Hoàng and Robin verified the conjecture for $t = 4$. In this talk, we confirm the conjecture for all $t \geq 4$. The proof depends on two newly established results on cycle structures in tough graphs, which hold independent interest.

The Chvátal-Erdős condition and Hamiltonicity of Cartesian products

Saturday
9:00-9:25

Yixuan Huang

Vanderbilt University

There are many remarkable sufficient conditions for the existence of Hamiltonian cycles in graphs. The Chvátal-Erdős Theorem, regarding the independence number $\alpha(G)$ and the connectivity $\kappa(G)$, states that if a graph G on at least three vertices satisfies $\alpha(G) \leq \kappa(G)$, then G has a Hamiltonian cycle. As an extension, Ellingham and Salehi Nowbandegani proved that if $\alpha(G) \leq 2\kappa(G)$, every such graph G is prism-Hamiltonian, that is, the Cartesian product of G and K_2 is Hamiltonian, and proved that $\alpha(G) \leq (t-1)\kappa(G)$ implies the Hamiltonicity of $G\Box C_t$ and thus the Hamiltonicity of $G\Box K_t$. They asked whether $\alpha(G) \leq t\kappa(G)$ implies the Hamiltonicity of $G\Box K_t$. We address this question by showing that $\alpha(G) \leq t\kappa(G)$ implies the Hamiltonicity of $G\Box C_t$.

This is joint work with Mark Ellingham, Pouria Salehi Nowbandegani, Songling Shan and Simon Špacapan.

Unavoidable immersions in finite and infinite graphs

Saturday
9:30-9:55

Brittian Qualls

Louisiana State University

The operation of lifting a pair of adjacent edges xy, yz is defined as deleting xy, yz and adding the edge xz. A graph H is called an immersion of a graph G if H can be obtained from a subgraph of G by repeated liftings. In this talk, we first discuss Ramsey-type "unavoidable" theorems in general, discussing some known results. Then we will describe our own results on finite graphs, which include the unavoidable immersions of 4-edge-connected graphs and some related implications on line graphs. Finally, we will shift our attention to infinite immersions, providing some background and finding the unavoidable immersions of infinite 3-edge-connected graphs. In fact, we provide something stronger, listing the unavoidable topological minors of infinite 3-edge-connected graphs.

Saturday
10:00-10:25

A note on bipartite graphs with the double Hall property

Yuying Ma

Georgia State University

Given a bipartite graph $G = (X, Y)$ and a subset $S \subseteq X$, we define $N_G^i(S)$ as the set of vertices in Y adjacent to at least i vertices in S. The graph G satisfies the *double Hall property (dHp)* if for every subset $S \subseteq X$ with $|S| \geq 2$, $|N_G^2(S)| \geq |S|$. A graph G is *supercyclic* if, under the same condition on S, there exists a cycle C_S in G such that $V(C_S) \cap X = S$. Obviously, every supercyclic graph satisfies dHp. In 2021, Nika Salia conjectured that the converse is also true.

In this paper, we investigate Salia's conjecture from several perspectives. We propose a new conjecture based on Salia's, requiring every vertex in Y to have a degree of at least $|X| - k$, with $|X|$ sufficiently large, and confirm it for $k \leq 7$. However, we observe that this conjecture is nearly equivalent to Salia's original one. Inspired by the work of Barát et al., we also prove Salia's conjecture for graphs where vertices in Y have degrees either 2 or very high. Finally, we give a lower bound on the maximum degree in dHp graphs and provide some constructions that nearly achieve this bound.

This is joint work with Guantao Chen, Mikhail Lavrov, Yimo Su, and Jennifer Vandenbussche.

Saturday
10:30-10:55

First-order model checking on monadically stable graph classes

Ioannis Eleftheriadis

University of Cambridge

A central goal in structural and algorithmic graph theory is to characterise the hereditary classes of structures that admit tractable first-order model checking. Here, it is conjectured that the model-theoretic notion of monadic dependence serves as the dividing line to tractability. In this work, we provide a positive answer to this conjecture in the context of edge-stable classes of graphs, i.e. those avoiding large semi-induced half graphs. On the tractable side, we show that all monadically stable graph classes have almost linear neighbourhood complexity, and consequently obtain sparse neighbourhood covers and tractable model checking. To prove hardness, we provide a characterisation of monadically stable graph classes in terms of forbidden families of induced subgraphs, and show that the hereditary closure of every edge-stable forbidden family effectively interprets the class of all graphs. This is joint work with Jan Dreier, Nikolas Mahlmann, Rose McCarty, Michal Pilipczuk, and Szymon Torunczyk (FOCS 2024).

Supersaturation, enumeration and randomness

Saturday
11:00-11:25

Sean Longbrake

Emory University

Let H be a graph. It is well known that if H is not bipartite, for every $\varepsilon > 0$, there is a δ such that if $e(G) \geq (1+\varepsilon)\mathrm{ex}(n, H)$, then G has $\delta n^{v(H)}$ many copies of H. Such a statement is usually called a supersaturation result, and such tight results are generally unavailable for bipartite graphs and usually a weaker $e(G) \geq m \geq C\mathrm{ex}(n, H)$ regime is considered because of scarcity of known exact Turan numbers for bipartite graphs. Statements like this have been only proven for a handful of graphs such as cycles, complete bipartite graphs, cubes with a diagonal. A more refined supersaturation results, known as balanced supersaturation, is usually used in conjunction with the container method to obtain corresponding enumeration results (f.e. to give upper bounds on the number of H-free graphs) or for random Turan problem (finding the largest H-free subgraph in $G_{n,p}$).

Recently we have established new general balanced supersaturation results for all bipartite graphs, and some tight bounds for certain tree-like graphs. In a separate work, we established similar balanced supersaturation results for certain hypergraph trees where every set has at least two leaves. Doing so we also achieve the corresponding enumeration result, as well as showing that with high probability, largest subgraph of $G_{n,p}(k)$ not containing T is at most the optimal $(1+o(1))p\mathrm{ex}(n, \mathcal{H})$ bound, when $p \gg \log(n)^2 n^{-k+3}$. In another work, we considered supersaturation problems for posets. For tree posets T, we count the number of subsets of the n-dimensional boolean lattice, showing that it is in fact the optimal $2^{(h(T)-1+o(1))\binom{n}{n/2}}$, as well as achieving asymptotically sharp results in the random setting. In this talk we will survey all three results, and methods that have been used to establish them, as well as give the state of art known in the literature.

The talk is based on various joint works with Tao Jiang, Sam Spiro, and Liana Yepremyan.

Saturday
11:30-11:55

The number of independent transversals in multipartite graphs

Yantao Tang

Georgia State University

We show that in any r-partite graph with parts of size at least n and maximum degree $\Delta < rn/(2r-2) - t$, the number of independent transversals is $\Omega(tn^{r-1})$. This generalizes a result of Jin for the $r = 4$ case and confirms a conjecure of Haxell and Szabó.

This talk is based on joint work with Yi Zhao.

Session II

Schedule

Saturday Afternoon		
Time	**Speaker**	**Title**
13:30-14:20	Xiaoyu He	Many Egyptian fractions equal one
14:30-14:55	Mikhail Lavrov	Online Ramsey numbers of ordered paths and cycles
15:00-15:50	Paul Seymour	Excluding a tree
20-min Coffee Break		
16:10-17:00	Yanli Hao	A proof of two conjectures on coloring multigraphs
17:10-17:35	Xiaofeng Gu	Spectral expansion properties of pseudorandom bipartite graphs

Many Egyptian fractions equal one

Saturday
13:30-14:20

Xiaoyu He

Georgia Institute of Technology

Let $f(n)$ denote the number of ways to write 1 as a sum of unit fractions $1/k$ where the denominators k are distinct natural numbers at most n. Answering a question of Erdős and Graham, we show that $f(n) = 2^{(c+o(1))n}$ for an explicit constant $c = .9117...$ The proof uses the entropy method and a simple form of iterative absorption.

Saturday
14:30-14:55

Online Ramsey numbers of ordered paths and cycles

Mikhail Lavrov
Kennesaw State University

An ordered graph is a graph with a linear ordering on its vertices. The Ramsey game for ordered graphs G and H is played on an infinite sequence of vertices; Builder draws an edge between two vertices, and Painter colors it red or blue. Builder tries to create a red G or a blue H as quickly as possible, while Painter wants the opposite.

When G and H are both ordered paths, a strategy for Builder is related to a sorting problem: the problem of finding a monotone subsequence guaranteed by the Erdős–Szekeres theorem. We look at what happens when G is a long ordered path and H is fixed; it turns out that there are several outcomes for how long the Ramsey game will last, depending on H, and we describe where the transition occurs.

This is joint work with Felix Clemen and Emily Heath.

Saturday
15:00-15:50

Excluding a tree

Paul Seymour
Princeton University

The Gyárfás-Sumner conjecture (from 1975, and still open) says that for every integer t and every tree T, if a graph has no t-vertex clique and has no induced subgraph isomorphic to T (briefly, is T-free), then its chromatic number is bounded.

If a graph has bounded chromatic number, it has a stable set of linear size; so it ought to be true that every K_t-free, T-free graph has a stable set of linear size, if we believe the Gyárfás-Sumner conjecture. This is quite different from what happens if we don't exclude a tree; for instance, K_{100}-free graphs G need not have stable sets of size more than $|G|^{1/49}$, which is far from linear. So excluding the tree T as well is supposed to make a lot of difference.

In joint work with Tung Nguyen and Alex Scott, we can prove that excluding T as well does make a lot of difference; if G is K_t-free and T-free where T is a tree, then G has a "nearly-linear" stable set.

We give a sketch of the proof, and survey some related results.

A proof of two conjectures on coloring multigraphs

Saturday
16:10-17:00

Yanli Hao

Georgia Institute of Technology

For a multigraph G, let $\chi'(G)$ denote the chromatic index of G, let $\Delta(G)$ the maximum degree of G, and let $\Gamma(G) = \max\{\lceil 2|E(H)|/(|V(H)|-1)\rceil : H \subseteq G \text{ and } |V(H)| \text{ odd}\}$. As a generalization of Vizing's classical coloring result for simple graphs, the Goldberg-Seymour conjecture, posed around 1973, asserts that $\chi'(G) = \max\{\Delta(G), \Gamma(G)\}$ or $\chi'(G) = \max\{\Delta(G)+1, \Gamma(G)\}$. Hochbaum, Nishizeki, and Shmoys also conjectured in 1986 that a $\max\{\Delta(G)+1, \chi'(G)\}$-edge-coloring of G can be found in polynomial time in the size of G. A long proof of the Goldberg-Seymour conjecture was announced in 2019 by Chen, Jing, and Zang, and one case in that proof was eliminated recently by Jing (but the proof is still long); and neither proof has been independently verified. In this paper, we give a proof of the Goldberg-Seymour conjecture that is significantly shorter and confirm the Hochbaum-Nishizeki-Shmoys conjecture by finding, in $O(|V(G)|^6|E(G)|^3)$ time, a $\max\{\Delta(G)+1, \Gamma(G)\}$-edge-coloring of G through Kempe changes.

Spectral expansion properties of pseudorandom bipartite graphs

Saturday
17:10-17:35

Xiaofeng Gu

University of West Georgia

An (a, b)-biregular bipartite graph is a bipartite graph with bipartition (X, Y) such that each vertex in X has degree a and each vertex in Y has degree b. By the bipartite expander mixing lemma, biregular bipartite graphs have nice pseudorandom and expansion properties when the second largest adjacency eigenvalue is not large. In this talk, several explicit properties of biregular bipartite graphs from spectral perspectives will be presented.

Session III

Schedule

*Light refreshments are available.

Sunday Morning		
Time	**Speaker**	**Title**
8:00-8:25	Yuval Wigderson	Finding large blowups
8:30-8:55	Vaidyanathan Sivaraman	Towards forbidden induced subgraphs of line graphs of directed graphs
9:00-9:50	Jessica McDonald	Strong Colouring with K_3's and K_4's
10:00-10:50	Paul Seymour	Recent progress on the Erdős-Hajnal conjecture
11:00-11:25	Corrine Yap	Dynamical thresholds for the fixed-magnetization Ising model
11:30-12:20	Yi Zhao	Extremal results in multipartite graphs

Finding large blowups

Sunday
8:00-8:25

Yuval Wigderson
ETH Zürich

A fundamental theorem of Nikiforov states that if a graph G contains many copies of a fixed graph H, then it contains a large blowup of H. In this talk, I will discuss this theorem, some of its applications, as well as a recent result which determines the optimal quantitative dependencies in case H is triangle-free.

Joint work with António Girão and Zach Hunter.

Sunday
8:30-8:55

Towards forbidden induced subgraphs of line graphs of directed graphs

Vaidyanathan Sivaraman
Mississippi State University

The line graph of a directed graph D is the graph whose vertex set is the edge set of D and two vertices are adjacent if the corresponding edges form a two-edge directed path in D. We discuss progress in finding the set of forbidden induced subgraphs for the class of line graphs of directed graphs. This is joint work with Daniel Slilaty.

Sunday
9:00-9:50

Strong colouring with K_3's and K_4's

Jessica McDonald
Auburn University

If H is a graph, and G is obtained from H by gluing on vertex-disjoint copies of K_t, then when can we guarantee that G is t-colourable? The *Strong Colouring Conjecture* posits that $\chi(G) \leq t$ whenever $t \geq 2\Delta(H)$. We'll discuss this seemingly very difficult conjecture, with particular focus on the elusive case of $\Delta(H) = 2$. We'll describe new joint work with Dalal and Shan where the "cycles plus K_4's" problem is reduced to a problem about "strong colouring" with K_3's.

Sunday
10:00-10:50

Recent progress on the Erdős-Hajnal conjecture

Paul Seymour
Princeton University

The Erdős-Hajnal conjecture (from 1977, and still open) says that for every graph H, there exists $c > 0$ such that in every H-free graph G there is a clique or a stable set of size at least $|G|^c$. This, with Hadwiger's conjecture, are perhaps the two most well-known and most studied conjectures in graph theory. This talk is a survey of what is known about the conjecture, including sketches of some recent results on it due to Tung Nguyen, Alex Scott and the speaker.

Dynamical thresholds for the fixed-magnetization Ising model

Saturday
11:00-11:25

Corrine Yap

Georgia Institute of Technology

Spin models on graphs are a source of many interesting questions in statistical physics, algorithms, and combinatorics. The Ising model is a classical example—first introduced as a model of magnetization, it can combinatorially be described as a weighted probability distribution on 2-vertex-colorings of a graph. We'll consider a fixed-magnetization version of the Ising model—akin to fixing the number of, say, blue vertices in every coloring—and a natural Markov chain sampling algorithm called the Kawasaki dynamics. We show some surprising results regarding the existence and location of a fast/slow mixing threshold for these dynamics. Our proofs require a combination of Markov chain tools, such as path coupling and spectral independence, with combinatorial tools, such as random graph analysis and second moment methods. Joint work with Aiya Kuchukova, Marcus Pappik, and Will Perkins.

Extremal results in multipartite graphs

Sunday
11:30-12:20

Yi Zhao

Georgia State University

Classical extremal results in graph theory (such as Turán's theorem) concern the maximal size of of a graph of given order and without certain subgraphs. Bollobás, Erdős, and Szemerédi in 1975 studied extremal problems in multipartite graphs. One of their problems (in its complementary form) was determining the maximal degree of a multipartite graph without an independent transversal. This problem has received considerable attention and was settle in 2006 by Szabó and Tardos and Haxell and Szabó. Other questions asked by Bollobás, Erdős, and Szemerédi remain open, such as determining

- the maximum degree in a multipartite graph without a *partial* independent transversal, and
- the minimum degree that forces an octahedral graph in balanced tripartite graphs.

In this talk I will survey recent progress on these and other related problems.

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