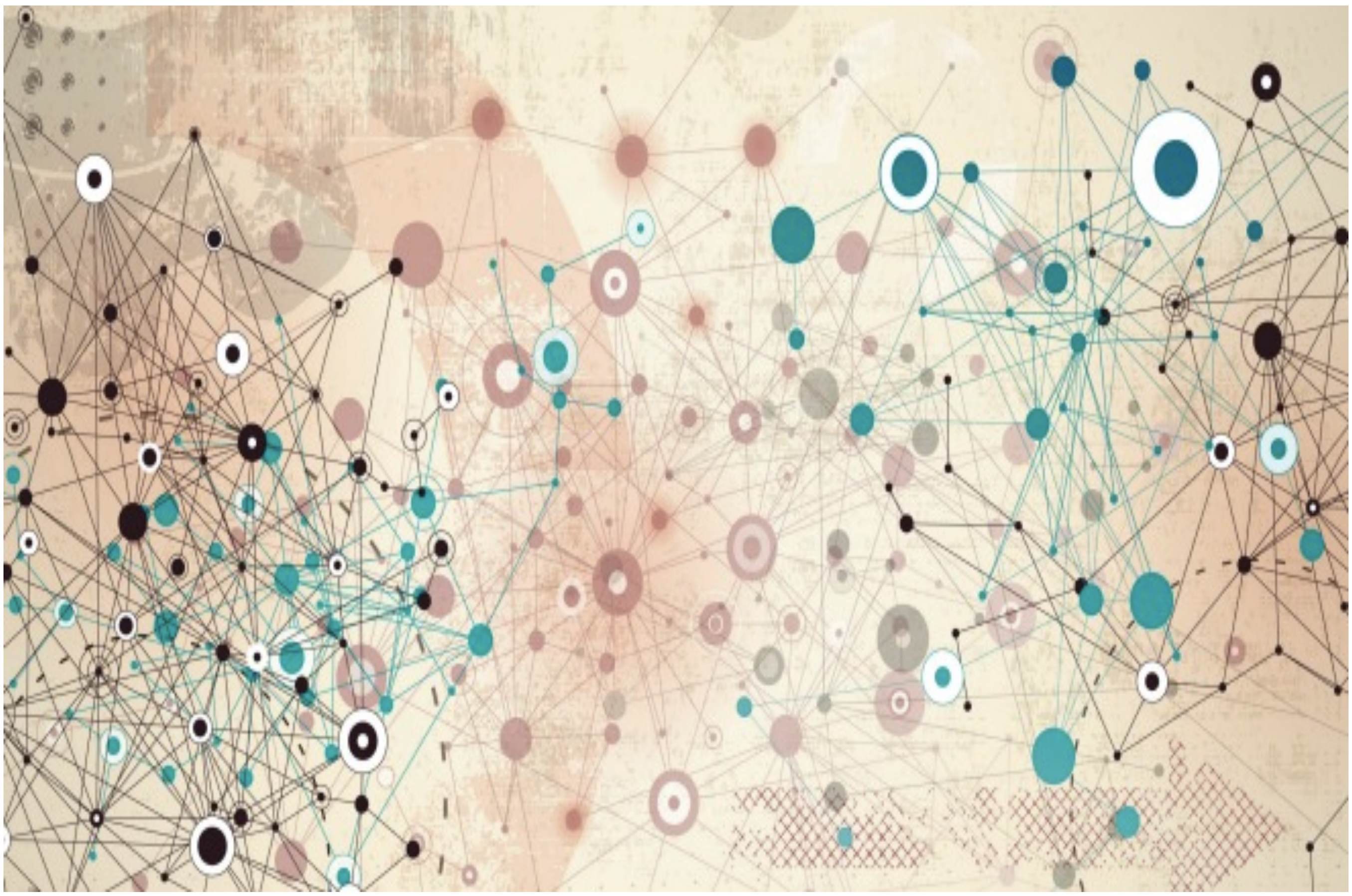
ATLANTA LECTURE SERIES

In Combinatorics and Graph Theory (XXVII)

November 4-5, 2023

GEORGIA STATE UNIVERSITY

Department of Mathematics and Statistics



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Mathematics and Statistics

Atlanta Lecture Series in Graph Theory and Combinatorics XXVII

Introduction

Atlanta Lecture Series (ALS) in Combinatorics and Graph Theory is a major event for the combinatorics community in the southeast region and beyond, providing opportunities to strengthen collaborations among researchers and institutions within the southeast region of the United States.

ALS has been alternately hosted by Emory University, Georgia Institute of Technology and Georgia State University, three major research universities in Atlanta metropolitan, before the COVID-19 pandemic, and currently been hosted by Georgia Institute of Technology and Georgia State University. This conference series is supported by the National Science Foundation. It features the latest research developments and themes in the areas of structural graph theory, extremal graph theory, random graphs, hypergraphs, and so on. Each Mini-Conference features one principle speaker and several other outstanding combinatorics/graph theorists, as well as some promising young researchers.

Speakers

The Featured Speaker

- Richard Montgomery, University of Warwick

One-hour Speakers

- Tom Kelly, Georgia Institute of Technology;
- Songling Shan, Auburn University;
- Liana Yepremyan, Emory University;
- Gexin Yu, College of William and Mary.

Half-hour Speakers

- Joseph Briggs, Auburn University;
- Yanli Hao, Georgia Institute of Technology;
- Zach Walsh, Georgia Institute of Technology;
- Jing Yu, Georgia Institute of Technology.

Schedule

Saturday Afternoon		
Time	**Speaker**	**Title**
13:00-13:50	Tom Kelly	Robustness for Hypergraph Embeddings via Spreadness
14:00-14:50	Richard Montgomery	On the Ryser-Brualdi-Stein Conjecture I
15:00-15:25	Zach Walsh	Planar Turán Number of the 7-cycle
15:30-15:55	Jing Yu	Large-scale Geometry of Graphs of Polynomial Growth
16:00-16:50	Songling Shan	Towards the Overfull Conjecture
Sunday Morning		
Time	**Speaker**	**Title**
8:30-9:20	Gexin Yu	Strong Edge-coloring of Graphs
9:30-10:20	Richard Montgomery	On the Ryser-Brualdi-Stein Conjecture II
10:30-10:55	Joseph Briggs	Looms
11:00-11:25	Yanli Hao	On the Coequal Values of Total Chromatic Number and Chromatic Index
11:30-12:20	Liana Yepremyan	Ramsey Graphs and Additive Combinatorics without Addition

Abstracts (in the order of the schedule)

Robustness for Hypergraph Embeddings via Spreadness

Tom Kelly

In this talk, we will discuss *robustness* results which lie in the intersection of both extremal and probabilistic combinatorics.

In joint work with Kang, Kühn, Methuku, and Osthus, we proved the following: If $p \geq C \log^2 n/n$ and $L_{i,j} \subseteq [n]$ is a random subset of $[n]$ where each $k \in [n]$ is included in $L_{i,j}$ independently with probability p for each $i, j \in [n]$, then asymptotically almost surely there is an order-n Latin square in which the entry in the ith row and jth column lies in $L_{i,j}$. We prove analogous results for Steiner triple systems and 1-factorizations of complete graphs. These results can be understood as stating that these “design-like” structures exist “robustly”.

In joint work with Kang, Kühn, Osthus, and Pfenninger, we proved various results stating that if $\mathcal{H}$ is an n-vertex k-uniform hypergraph satisfying some minimum degree condition and $p = \Omega(n^{-k+1} \log n)$, then asymptotically almost surely a p-random subhypergraph of $\mathcal{H}$ contains a perfect matching. These results can be understood as “robust” versions of hypergraph Dirac-type results as they simultaneously strengthen Johansson, Kahn, and Vu’s seminal solution to Shamir’s problem on the threshold for when a binomial random k-uniform hypergraph contains a perfect matching. In joint work with Müyesser, and Pokrovskiy, we proved similar results for hypergraph Hamilton cycles.

All of these results utilize the recent Park—Pham Theorem or one of its variants. A crucial notion for this is that of the *spreadness* of a certain type of probability distribution.

On the Ryser-Brualdi-Stein Conjecture

Richard Montgomery

A Latin square of order n is an n by n grid filled with n symbols, so that every symbol appears exactly once in each row and each column. A partial transversal of a Latin square of order n is a collection of cells in the grid which share no row, column or symbol, while a full transversal is a transversal with n cells.

The Ryser-Brualdi-Stein conjecture states that every Latin square of order n should have a partial transversal with n-1 elements, and a full transversal if n is odd. In 2020, Keevash, Pokrovskiy, Sudakov and Yepremyan improved the long-standing best known bounds on this conjecture by showing that a partial transversal with $n - O(\log n / \log \log n)$ elements always exists.

I will discuss how to show, for large n, that a partial transversal with $n - 1$ elements always exists. The method has several key elements, allowing the discussion of the proof in each talk to be largely self-contained.

Planar Turán Number of the 7-cycle

Zach Walsh

The planar Turán number of a graph H is the maximum number of edges in an n-vertex planar graph without H as a subgraph. The planar Turán number of a length-t cycle is known when $t = 3, 4, 5, 6$, and two independent groups recently found the planar Turán number of the length-7 cycle. We will discuss some proof techniques and related open problems. This is joint work with Ruilin Shi and Xingxing Yu.

Large-scale Geometry of Graphs of Polynomial Growth

Jing Yu

In 1995, Levin and Linial, London, and Rabinovich conjectured that every connected graph G of polynomial growth admits an injective homomorphism to the n-dimensional grid graph for some n. Moreover, they conjected that if every ball of radius r in G contains at most $O(r^{\rho})$ vertices, then one can take $n = O(\rho)$. Krauthgamer and Lee confirmed the first part of this conjecture and refuted the second in 2007. By constructing some finite expander graphs, they showed best possible upper bound on n is $O(\rho \log \rho)$. Prompted by these results, Papasoglu asked whether a graph G of polynomial growth admits a coarse embedding into a grid graph. We give an affirmative answer to this question. Moreover, it turns out that the dimension of the grid graph only needs to be linear in the asymptotic growth rate of G, which confirms the original Levin–Linial–London–Rabinovich conjecture "on the large scale." Besides, we find an alternative proof of the result of Papasoglu that graphs of polynomial growth rate $\rho < \infty$ have asymptotic dimension at most ρ. Furthermore, our proof works in the Borel setting and shows that Borel graphs of polynomial growth rate $\rho < \infty$ have Borel asymptotic dimension at most ρ. This is joint work with Anton Bernshteyn.

Towards the Overfull Conjecture

Songling Shan

Let G be a simple graph with maximum degree denoted as $\Delta(G)$. An overfull subgraph H of G is a subgraph satisfying the condition $|E(H)| > \Delta(G)\lfloor \frac{1}{2}|V(H)|\rfloor$. In 1986, Chetwynd and Hilton proposed the Overfull Conjecture, stating that a graph G with maximum degree $\Delta(G) > \frac{1}{3}|V(G)|$ has chromatic index equal to $\Delta(G)$ if and only if it does not contain any overfull subgraph. The Overfull Conjecture has many implications. For example, it implies a polynomial-time algorithm for determining the chromatic index of graphs G with $\Delta(G) > \frac{1}{3}|V(G)|$, and implies several longstanding conjectures in the area of graph edge colorings. In this paper, we make the first improvement towards the conjecture when not imposing a minimum degree condition on the graph: for any $0 < \varepsilon \leq \frac{1}{22}$, there exists a positive integer n_0 such that if G is a graph on $n \geq n_0$ vertices with $\Delta(G) \geq (1-\varepsilon)n$, then the Overfull Conjecture holds for G. The previous best result in this direction, due to Chetwynd and Hilton from 1989, asserts the conjecture for graphs G with $\Delta(G) \geq |V(G)| - 3$.

Strong Edge-coloring of Graphs

Gexin Yu

A strong edge-coloring of a graph G is proper edge-coloring of G such that each color class forms an induced matching. Erdős and Nestril conjectureed that every graph with maximum degree Δ has a strong edge-coloring with at most $1.25\Delta^2$ colors. This conjecture is verified for $\Delta = 3$ and remains open for $\Delta \geq 4$. In this talk, we will survey some recent progress on strong edge-coloring, and verify a related conjecture by Hocquard, Lajou, and Lužar that every subcubic graph can be decomposed into two matchings and four induced matchings.

Looms

Joseph Briggs

A ridiculous conjecture of Gyárfás-Lehel suggests an improvement of 1 on a trivial bound of the number of vertices needed to cover every hyperedge in an r-partite pair of cross-intersecting hypergraphs. We don't know how to solve this conjecture. But we can reduce it to showing every pair of uniform hypergraphs that are mutually the set of each other's minimum covers, which we call a loom, has a perfect fractional matching in each part. Alas, looms appear enigmatic to construct. So we show that all looms that we *can* build satisfy this conjecture.

On the Coequal Values of Total Chromatic Number and Chromatic Index

Yanli Hao

The **chromatic index** $\chi'(G)$ of a graph G is the least number of colors assigned to the edges of G such that no two adjacent edges receive the same color. The **total chromatic number** $\chi''(G)$ of a graph G is the least number of colors assigned to the edges and vertices of G such that no two adjacent edges receive the same color, no two adjacent vertices receive the same color and no edge has the same color as its two endpoints. The chromatic index and the total chromatic number are two of few fundamental graph parameters, and their correlation has always been a subject of intensive study in graph theory.

By definition, $\chi'(G) \leq \chi''(G)$ for every graph G. In 1984, Goldberg conjectured that for any multigraph G, if $\chi'(G) \geq \Delta(G) + 3$ then $\chi''(G) = \chi'(G)$. We show that Goldberg's conjecture is asymptotically true. More specifically, we prove that for a multigraph G with maximum degree Δ sufficiently large, $\chi''(G) = \chi'(G)$ provided $\chi'(G) \geq \Delta + 10\Delta^{35/36}$. When $\chi'(G) \geq \Delta(G) + 2$, the chromatic index $\chi'(G)$ is completely determined by the fractional chromatic index. Consequently, the total chromatic number $\chi''(G)$ can be computed in polynomial-time in this case.

Ramsey Graphs and Additive Combinatorics without Addition

Liana Yepremyan

A graph is Ramsey if its largest clique or independent set is of size logarithmic in the number of vertices. While almost all graphs are Ramsey, there is still no known explicit construction of Ramsey graphs. We discuss recent work on some questions in this area. Along the way, we study some fundamental problems in additive combinatorics, and discover that group structure is superfluous for these problems. Joint work with David Conlon, Jacob Fox and Huy Tuan Pham.