Atlanta Lecture Series in Graph Theory and Combinatorics XXV

Introduction

Atlanta Lecture Series (ALS) in Combinatorics and Graph Theory is a major event for the combinatorics community in the southeast region and beyond, providing opportunities to strengthen collaborations among researchers and institutions within the southeast region of the United States.

ALS has been alternately hosted by Emory University, Georgia Institute of Technology, and Georgia State University, three major research universities in Atlanta metropolitan. This conference series is supported by the National Security Agency and the National Science Foundation. It features the latest research developments and themes in the areas of structural graph theory, extremal graph theory, random graphs, hypergraphs, and so on. Each Mini-Conference features one principle speaker and several other outstanding combinatorics/graph theorists, as well as some promising young researchers.

Speakers

The Featured Speaker

- Rob Morris, IMPA (Instituto Nacional de Matemàtica Pure e Applicada)

One-hour Speakers

- Hal Kierstead, Arizona State University
- Lincoln Lu, University of South Carolina
- Vladimir Nikiforov, University of Memphis
- Songling Shan, Illinois State University

Half-hour Speakers

- Alexander Clifton, Emory University
- Blake Dunshee, Vanderbilt University
- He Guo, Georgia Institute of Technology
- Xujun Liu, University of Illinois Urbana-Champaign
- Vaidyanathan Sivaraman, Mississippi State University
- Zhiyu Wang, Georgia Institute of Technology
- Michael Wigal, Georgia Institute of Technology

- Dong Ye, Middle Tennessee State University

- Guoning Yu, Georgia State University

Schedule

Saturday Morning		
Time	**Speaker**	**Title**
9:00-11:55	***Morning Session***	
9:00-9:25	Xujun Liu	Packing Colorings of Subcubic Graphs
9:30-9:55	Alexander Clifton	Almost k-covers of Hypercubes
10:00-10:25	He Guo	Prague Dimension of Random Graphs
10:30-10:55	Blake Dunshee	Directed Embeddings with Partially Predetermined Facial Walks
11:00-11:25	Guoning Yu	Linear Arboricity of Degenerate Graphs
11:30-11:55	Michael Wigal	Long Cycles in Essentially 4-connected Projective-planar Graphs
Saturday afternoon		
Time	**Speaker**	**Title**
13:00-13:50	Hal Kierstead	Uniform Orderings for Generalized Coloring Numbers
14:00-14:50	Rob Morris	Erdős Covering Systems
14:50-15:20	**Break**	
15:20-16:10	Vladimir Nikiforov	Real Eigenvalues of Complete Hypergraphs and Extrema of Symmetric Functions
16:15-16:40	Dong Ye	Integer Flows and Sign-Circuit Covering
16:45-17:10	Vaidyanathan Sivaraman	Polynomial χ-boundedness

Sunday Morning		
Time	**Speaker**	**Title**
8:30-9:20	Lincoln Lu	Anti-Ramsey Number of Edge-disjoint Rainbow Spanning Trees in all Graphs
9:30-10:20	Rob Morris	Flat Littlewood Polynomials Exist
10:30-11:20	Songling Shan	Some Recent Progress Towards the Overfull Conjecture
11:30-11:55	Zhiyu Wang	Polynomial χ-binding functions for t-broom-free graphs

Abstracts

Packing Colorings of Subcubic Graphs

Xujun Liu

For a sequence of non-decreasing positive integers $S = (s_1, \ldots, s_k)$, a packing S-coloring is a partition of $V(G)$ into sets $V_1, \ldots, V_k$ such that for each $1 \leq i \leq k$ the distance between any two distinct $x, y \in V_i$ is at least $s_i + 1$. The smallest k such that G has a packing $(1, 2, \ldots, k)$-coloring is called the packing chromatic number of G and is denoted by $\chi_p(G)$. The question whether the packing chromatic number of subcubic graphs is bounded appeared in many papers. Another popular problem in the topic is the conjecture of Brešar, Klavžar, Rall and Wash that the packing chromatic number of subdivision of subcubic graphs are bounded above by 5.

With Balogh and Kostochka, we showed that for every fixed k and $g \geq 2k+2$, almost every n-vertex cubic graph of girth at least g has the packing chromatic number greater than k, which answers the previous question in the negative. In contrast, we proved the first and current best upper bound, 8, on the conjecture of Brešar et al. We will also talk about recent results which confirm the conjecture of Brešar et al for some subclasses of subcubic graphs.

Almost k-covers of Hypercubes

Alexander Clifton

Alon and Füredi showed that covering all vertices of an n-dimensional hypercube except one requires at least n hyperplanes. We extend this question to covering all vertices at least k times, except for one which is not covered at all. For every n and k, we solve the fractional case of this problem by generalizing the Lubell-Yamamoto-Meshalkin inequality. For small values of k, we use the Punctured Combinatorial Nullstellensatz of Ball and Serra to solve the problem for $k = 3$ and to establish a nontrivial lower bound when $k > 3$.

Joint work with Hao Huang.

Prague Dimension of Random Graphs

He Guo

The Prague dimension of graphs was introduced by Nesetril, Pultr and Rodl in the 1970s. Proving a conjecture of Furedi and Kantor, we show that the Prague dimension of the binomial random graph is typically of order $n/\log n$ for constant edge-probabilities. The main new proof ingredient is a Pippenger-Spencer type edge-coloring result for random hypergraphs with large uniformities, i.e., edges of size $O(\log n)$.

Based on joint work with Kalen Patton and Lutz Warnke.

Directed Embeddings with Partially Predetermined Facial Walks

Blake Dunshee

A directed embedding is a digraph embedded in a surface in such a way that all the faces of the embedding are bounded by directed walks. This is equivalent to an embedding of a digraph where half-arcs alternate between in and out around a vertex. First, we characterize when an embedded graph can be given an orientation such that the resulting embedding of a digraph is a directed embedding. We then characterize when a digraph and a collection of its closed directed walks can be extended to a directed embedding such that the collection of closed directed walks is a subcollection of the facial walks. Furthermore, we characterize when such a directed embedding can be chosen to be orientable. Širáň and Škoviera gave analogous results in the case of undirected graphs by characterizing when a graph and a collection of its closed walks can be extended to an embedding such that the collection of closed walks is a subcollection of the facial walks. The necessary and sufficient conditions are stronger in the case of orientable directed embeddings than those for orientable embeddings of undirected graphs.

Linear Arboricity of Degenerate Graphs

Guoning Yu

A linear forest is a union of vertex-disjoint paths, and the linear arboricity of a graph G, denoted by $\mathrm{la}(G)$, is the minimum number of linear forests needed to partition the edge set of G. Let G be a graph with maximum degree $\Delta(G)$. Clearly, $\mathrm{la}(G) \geq \lceil \Delta(G)/2 \rceil$. On the other hand, the famous Linear Arboricity Conjecture (LAC) due to Akiyama, Exoo, and Harary from 1981 asserts that $\mathrm{la}(G) \leq \lceil (\Delta(G)+1)/2 \rceil$. The conjecture has been verified for very special graphs such as planar graphs and graphs with maximum degree up to $6, 8$ and 10.

A graph G is k-degenerate for a positive integer k if it can be reduced to a trivial graph by successive removal of vertices with degree $\leq k$. In this talk, I will describe a proof of $\mathrm{la}(G) = \lceil \Delta(G)/2 \rceil$ for any k-degenerate graph G with $\Delta(G) \geq 2k^2 - k$.

Joint work with Guantao Chen and Yanli Hao.

Long Cycles in Essentially 4-connected Projective-planar Graphs

Michael Wigal

Tutte paths have a critical role in the study of Hamiltonicity for 4-connected planar and other graph classes. We show quantitative Tutte path results in which we bound the number of bridges of the path. A corollary of this result is near optimal circumference bounds for essentially 4-connected planar and projective-planar graphs.

Joint work with Xingxing Yu.

Uniform Orderings for Generalized Coloring Numbers

Hal Kierstead

Nešetřil and Ossona de Mendez introduced the notion of graph classes *with bounded expansion* and the more general notion of *nowhere-dense* graph classes. These concepts generalize those of graph classes with bounded tree-width, minor-closed classes, bounded degree classes, etc. This classification is informative as many interesting properties of simple sparse classes are shared with more general classes, while results on the general classes can often be sharpened for simpler classes. Moreover, the Nešetřil-Ossona de Mendez formulation is remarkably robust; there are many apparently disparate notions that turn out to be equivalent.

In applications it is often useful to use characterizations due to Zhu of bounded-expansion or nowhere-dense classes in terms of the *generalized coloring numbers* $\mathrm{scol}_r(G)$ of a graph G. These numbers had been introduced earlier by Yang and me to extend the classes of graphs known to have bounded generalized *game* coloring numbers. Zhu's result implies that these are exactly the classes with bounded expansion. For each distance r, the strong r-coloring number $\mathrm{scol}_r(G)$ is determined by an "optimal" ordering of the vertices of G. We study the question of whether it is possible to find a single "uniform" ordering that is "good" for all distances r. We show that the answer to this question is essentially "yes". Our results give new characterizations of bounded-expansion and nowhere-dense graph classes.

Much of this talk will be on joint work with Jan van den Heuvel, Department of Mathematics, London School of Economics and Political Science.

Erdős Covering Systems

Rob Morris

A *covering system* of the integers is a finite collection of arithmetic progressions whose union is the set $\mathbb{Z}$. The study of these objects was initiated in 1950 by Erdős, and over the following decades he asked a number of beautiful questions about them. Most famously, his so-called “minimum modulus problem" was resolved in 2015 by Hough, who proved that in every covering system with distinct moduli, the minimum modulus is at most 10^{16}.

In this talk I will describe a simple and general method of attacking covering problems that was inspired by Hough’s proof. We expect that this technique, which we call the “distortion method", will have further applications in combinatorics.

This talk is based on joint work with Paul Balister, Bela Bollobas, Julian Sahasrabudhe and Marius Tiba.

Real Eigenvalues of Complete Hypergraphs and Extrema of Symmetric Functions

Vladimir Nikiforov

Calculating, or even bounding the real eigenenvalues of uniform complete hypergraphs turns out to be a hard problem. In this talk we present the order of magnitude of real eigenvalues of complete uniform hypergraphs that are different from their spectral radius. Problems of this type have been raised recently in computer science and they amount to finding the critical points of the elementary symmetric functions over unit spheres in the l^p norm.

Integer Flows and Sign-Circuit Covering

Dong Ye

Let (G, σ) be a signed graph. A sign-circuit of (G, σ) is a minimal dependent set of its signed graphic matroid, which is either a positive cycle or a barbell (two negative cycles joined by a path). Bouchet proved that a signed graph has a sign-circuit cover if and only if it has a nowhere-zero integer flow. However, most properties on circuit cover and integer flows for graphs fail for signed graphs. In this talk, we focus some connections between integer flows and sign-circuit covering, and recent improvement on shortest sign-circuit covering. This talk is based on joint work with Jiaao Li and Yezhou Wu.

Polynomial χ-boundedness

Vaidyanathan Sivaraman

For what graph classes can we bound the chromatic number of a graph as a polynomial function of its clique number? Such a graph class is called polynomially χ-bounded. This talk will survey graph classes which are known to be polynomially χ-bounded. Several open problems will also be mentioned.

Anti-Ramsey Number of Edge-disjoint Rainbow Spanning Trees in all Graphs

Lincoln Lu

An edge-colored graph H is called *rainbow* if every edge of H receives a different color. Given any host multigraph G, the *anti-Ramsey* number of t edge-disjoint rainbow spanning trees in G, denoted by $r(G,t)$, is defined as the maximum number of colors in an edge-coloring of G containing no t edge-disjoint rainbow spanning trees. For any vertex partition P, let $E(P,G)$ be the set of non-crossing edges in G with respect to P. In this talk, we determine $r(G,t)$ for all host multigraphs G: $r(G,t) = |E(G)|$ if there exists a partition P_0 with $|E(G)| - |E(P_0,G)| < t(|P_0|-1)$; and $r(G,t) = \max_{P:\,|P|\geq 3}\{|E(P,G)| + t(|P|-2)\}$ otherwise. As a corollary, we determine $r(K_{p,q},t)$ for all values of p,q,t, improving a result of Jia, Lu and Zhang. (Joint work with Andrew Meier and Zhiyu Wang)

Flat Littlewood Polynomials Exist

Rob Morris

In a Littlewood polynomial, all coefficients are either 1 or -1. Littlewood proved many beautiful theorems about these polynomials over his long life, and in his 1968 monograph he stated several influential conjectures about them. One of the most famous of these was inspired by a question of Erdős, who asked in 1957 whether there exist “flat" Littlewood polynomials of degree n, that is, with $|P(z)|$ of order $n^{1/2}$ for all (complex) z with $|z| = 1$.

In this talk I will describe a proof that flat Littlewood polynomials of degree n exist for all $n > 1$. The proof is entirely combinatorial, and uses probabilistic ideas from discrepancy theory.

Based on joint work with Paul Balister, Béla Bollobás, Julian Sahasrabudhe and Marius Tiba.

Some Recent Progress Towards the Overfull Conjecture

Songling Shan

Let G be a simple graph with maximum degree $\Delta(G)$. A subgraph H of G is overfull if $|E(H)| > \Delta(G)\lfloor |V(H)|/2 \rfloor$. Chetwynd and Hilton in 1985 conjectured that a graph G with $\Delta(G) > |V(G)|/3$ has chromatic index $\Delta(G)$ if and only if G contains no overfull subgraph. In this talk, we will survey some recent progress towards the conjecture. In particular, we will mention the confirmation of the conjecture on graphs with a small core degree, dense quasirandom graphs of odd order, and large graphs of order $2n$ and minimum degree at least $(1+\varepsilon)n$ for any $0 < \varepsilon < 1$.

Polynomial χ-binding functions for t-broom-free graphs

Zhiyu Wang

For any positive integer t, a *t-broom* is a graph obtained from $K_{1,t+1}$ by subdividing an edge once. In this paper, we show that, for graphs G without induced t-brooms, we have $\chi(G) = o(\omega(G)^{t+1})$, where $\chi(G)$ and $\omega(G)$ are the chromatic number and clique number of G, respectively. When $t = 2$, this answers a question of Schiermeyer and Randerath. Moreover, for $t = 2$, we strengthen the bound on $\chi(G)$ to $7.5\omega(G)^2$, confirming a conjecture of Sivaraman. For $t \geq 3$ and {t-broom, $K_{t,t}$}-free graphs, we improve the bound to $o(\omega^{t-1+\frac{2}{t+1}})$. Joint work with Xiaonan Liu, Joshua Schroeder and Xingxing Yu.