Atlanta Lecture Series in Graph Theory and Combinatorics XXII

Introduction

Atlanta Lecture Series (ALS) in Combinatorics and Graph Theory is a major event for the combinatorics community in the southeast region and beyond, providing opportunities to strengthen collaborations among researchers and institutions within the southeast region of the United States.

This conference series is supported by the National Security Agency and the National Science Foundation. It features the latest research developments and themes in the areas of structural graph theory, extremal graph theory, random graphs, hypergraphs, and so on. Each Mini-Conference features one principle speaker and several other outstanding combinatorics/graph theorists, as well as some promising young researchers.

Speakers

The Featured Speaker

- Mathias Schacht, Universität Hamburg, Germany & Yale University, USA

One-hour Speakers

- Guoli Ding, Louisiana State University, Baton Rouge,USA
- Ronald Gould, Emory University, Atlanta, USA
- Guangming Jing, Georgia State University, Atlanta, USA
- Wojciech Samotij, Tel Aviv University, Tel Aviv, Israel

Half-hour Speakers

- Jie Han, University of Rhode Island, Kingston, USA
- Dong Quan Ngoc Nguyen, University of Notre Dame, Notre Dame, USA
- Gregory Puleo, Auburn University, Auburn, USA
- Songling Shan, Illinois State University, Normal, USA
- Ryan Solava, Vanderbilt University, Nashville, USA

Schedule

Time (Saturday)	**Speaker**	**Title**
9:30am-12:05pm		**Mini-workshop**
12:05pm-12:30pm	**Break**	
12:30pm-12:55pm	Gregory Puleo	t-cores for $(\Delta + t)$-edge-colouring
1:00pm-1:50pm	Wojciech Samotij	Subsets of Posets Minimising the number of Chains
2:00pm-2:50pm	Guangming Jing	The Goldberg-Seymour Conjecture on Edge-Colorings of Multigraphs
2:50pm-3:20pm	**Break**	
3:20pm-4:10pm	Mathias Schacht	Hamiltonian cycles in Hypergraphs
4:15pm-4:45pm	Songling Shan	Circumferences of 3-connected graphs with bounded maximum degrees
4:50pm-5:20pm	Dong Quan Ngoc Nguyen	Distribution of certain generalized binomial coefficients

Time (Sunday)	**Speaker**	**Title**
8:30am-8:55am	Jie Han	The Absorption Technique in Graphs and Hypergraphs
9:00am-9:50am	Guoli Ding	A Survey on Infinite Antichains of Graphs
9:55am-10:45am	Mathias Schacht	Powers of Hamiltonian cycles in randomly augmented graphs
10:50am-11:40am	Ronald Gould	A Look at Saturated Graphs
11:45am-12:15pm	Ryan Solava	Fine Structure of 3-connected $K_{2,t}$-minor-free Graphs

Abstracts

t-cores for $(\Delta + t)$-edge-colouring

Gregory J. Puleo

A foundational theorem in edge colouring is Vizing's Theorem, which states that if G is a multigraph with maximum degree $\Delta(G)$ and maximum multiplicity $\mu(G)$, then G can be properly edge-coloured with $\Delta(G) + \mu(G)$ colours, that is, $\chi'(G) \leq \Delta(G) + \mu(G)$. In this talk, we will discuss conditions under which this upper bound can be improved.

The conditions we consider are based on the idea of the *core* of a graph. The core of a simple graph G is the subgraph induced by its vertices of maximum degree. A theorem of Fournier states that if G is a simple graph whose core is a forest, then $\chi'(G) = \Delta(G)$. This theorem was strengthened by Hoffman and Rodger, who proved that if the core of G admits a structure called a *full B-queue*, then $\chi'(G) = \Delta(G)$.

We define the *t-core* of a multigraph G, for integer $t \geq 0$, to be the subgraph induced by the vertices v such that $d(v) + \mu(v) > \Delta(G) + t$. (Thus, the 0-core of a simple graph is just its core, as defined above.) We use the *Fan number*, introduced by Scheide and Stiebitz, to obtain multigraph generalizations of theorems such as the theorems of Fournier and Hoffman–Rodger, using the notion of t-cores to replace the notion of cores. This is joint work with Jessica McDonald.

Subsets of posets minimising the number of chains

Wojciech Samotij

A well-known theorem of Sperner describes the largest collections of subsets of an n-element set none of which contains another set from the collection. Generalising this result, Erdős characterised the largest families of subsets of an n-element set that do not contain a chain of sets $A_1 \subset \ldots \subset A_k$ of an arbitrary length k. The extremal families contain all subsets whose cardinalities belong to an interval of length $k-1$ centred at $n/2$. In a far-reaching extension of Sperner's theorem, Kleitman determined the smallest number of chains of length two that have to appear in a collection of a given number a of subsets. For every a, this minimum is achieved by the collection comprising a sets whose cardinalities are as close to $n/2+1/4$ as possible. We show that the same is true about chains of an arbitrary length k, for all a and n, confirming the prediction Kleitman made fifty years ago.

The Goldberg-Seymour Conjecture on Edge-Colorings of Multigraphs

Guangming Jing

Given a multigraph $G = (V, E)$, the *edge-coloring problem* (ECP) is to color the edges of G with the minimum number of colors such that no two adjacent edges receive the same color. This problem can naturally be formulated as an integer program, and its linear programming relaxation is called the *fractional edge-coloring problem* (FECP). The optimal value of ECP (resp. FECP) is called the *chromatic index* (resp. *fractional chromatic index*) of G, denoted by $\chi'(G)$ (resp.$\chi^*(G)$). Let $\Delta(G)$ be the maximum degree of G and let $w(G) = \max_{H \subseteq G} \left\lceil \frac{|E(H)|}{\lfloor \frac{1}{2}|V(H)| \rfloor} \right\rceil$. Clearly, $\max\{\Delta(G), w(G)\}$ is a lower bound for $\chi'(G)$. As shown by Seymour, $\chi^*(G) = max\{\Delta(G), w(G)\}$. In the 1970s Goldberg and Seymour independently conjectured that $\chi'(G) \leq max\{\Delta(G) + 1, w(G)\}$, which if true implies that, first, every multigraph G satisfies $\chi'(G) - \chi^*(G) \leq 1$, so FECP enjoys a fascinating integer rounding property; second, ECP can be approximated within one of the optimum, and hence is one of the "easiest" NP-hard problems; third, there are only two possible values for $\chi'(G)$, so an analogue to Vizing's theorem on edge-colorings of simple graphs, a fundamental result in graph theory, holds for multigraphs. In this talk, we will discuss the proof of this conjecture.

This is a joint work with Guantao Chen and Wenan Zang.

Hamiltonian cycles in Hypergraphs

Mathias Schacht

Over the last two decades many extensions of Dirac's theorem to hypergraphs were obtained. We shall focus on 3-uniform hypergraphs and show that if every vertex is contained in at least $\frac{5}{9} + o(1)$ proportion of all possible edges, then it contains a (tight) Hamiltonian cycle. Several different lower bound constructions show that this degree condition is asymptotically optimal.

Circumferences of 3-connected graphs with bounded maximum degrees

Songling Shan

In 1993 Jackson and Wormald conjectured that for any positive integer d with $d \geq 4$, there exists a positive real number α depending only on d such that if G is a 3-connected n-vertex graph with maximum degree at most d, then G has a cycle of length at least $\alpha n^{\log_{d-1} 2}$. They showed that the exponent in the bound is best possible if the conjecture is true. We confirm the conjecture for $d \geq 370$.

Distribution of certain generalized binomial coefficients

Dong Quan Ngoc Nguyen

For a prime p and a positive integer n, a theorem of Garfield and Wilf can tell us the number of integers $0 \leq m \leq n$ for which the binomial coefficient $\binom{n}{m}$ falls into each of the residue classes $0, 1, \ldots, p-1$ (modulo p). In this talk, I will discuss a variant of this theorem in the function field setting, where integers are replaced by polynomials over a finite field. I will also talk about a generalization of Garfield and Wilf's theorem in which the ring of integers is replaced by a discrete valuation domain with finite residue field.

The absorption technique in graphs and hypergraphs

Jie Han

The study of subgraphs is a central topic in graph theory. There has been a lot of work on embedding spanning substructures, such as, perfect matchings, Hamiltonian cycles and clique-factors. We will introduce some recent developments on this line of research, in particular, via the powerful absorption technique.

A survey on infinite antichains of graphs

Guoli Ding

Let $\preceq$ be a graph containment relation, by which I mean relations like the subgraph relation, the minor relation, and so on. Then a (finite or infinite) set $\mathcal{A}$ of graphs is called a $\preceq$-*antichain* if $G \preceq H$ never holds for any two distinct graphs G and H of $\mathcal{A}$. For example, the set $\mathcal{C}$ of all cycles is a subgraph-antichain while $\mathcal{C}$ is not a minor-antichain. A celebrated result of Robertson and Seymour, now known as the *graph minor theorem*, states that every minor-antichain is finite. This far-reaching result motivates the following question: what can we say about the finiteness of other $\preceq$-antichains? The purpose of my talk is to survey partial answers to this question.

Powers of Hamiltonian cycles in randomly augmented graphs

Mathias Schacht

We study the existence of powers of Hamiltonian cycles in graphs with large minimum degree to which some additional edges have been added in a random manner. It follows from the theorems of Dirac and of Komlós, Sarközy, and Szemerédi that for every k and sufficiently large n already the minimum degree $\geq \frac{k}{k-1}n$ for an n-vertex graph G alone suffices to ensure the existence of a k-th power of a Hamiltonian cycle. We show that under essentially the same degree assumption the addition of just $O(n)$ random edges ensures the presence of the $(k+1)$-st power of a Hamiltonian cycle with probability close to one.

A Look at Saturated Graphs

Ronald J. Gould

Given a graph H, we say that a graph G is *H-saturated* if it does not contain H as a subgraph but the addition of any edge $e \notin E(G)$ results in at least one copy of H as a subgraph. The study of saturated graphs has a long and deep history. The question of the minimum number of edges of an H-saturated graph on n vertices, known as the *saturation number* and denoted $sat(n, H)$, has been addressed for many different types of graphs. The saturation number contrasts the classic question of the maximum number of edges possible in a graph G on n vertices that does not contain a copy of H, known as the *Turán number* (or *extremal number*) and denoted $ext(n, H)$. The *saturation spectrum* of the family of H-saturated graphs on n vertices is the set of all possible sizes $(|E(G)|)$ of an H-saturated.We look at a number of recent results that determine the saturation spectrum for a variety of graphs.

Fine structure of 3-connected $K_{2,t}$-minor-free graphs

Ryan Solava

In this talk, I will discuss two results that describe the structure of graphs that avoid certain minors. The first of these is a structural characterization of 3-connected $K_{2,5}$-minor-free graphs, which allows for an exact counting result. The second describes the structure of 3 and 4-connected $K_{2,t}$-minor-free graphs for any given t. This result gives the asymptotic counts for these families.