ATLANTA LECTURE SERIES

In Combinatorics and Graph Theory (XIX)

April 22-23, 2017

GEORGIA STATE UNIVERSITY

Department of Mathematics and Statistics

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Mathematics and Statistics

Introduction

Atlanta Lecture Series (ASL) in Combinatorics and Graph Theory is a major event for the combinatorics community in the southeast region and beyond, providing opportunities to strengthen collaborations among researchers and institutions within the southeast region of the United States.

This conference series is supported by the National Security Agency and the National Science Foundation. It features the latest research developments and themes in the areas of structural graph theory, extremal graph theory, random graphs, hypergraphs, and so on. Each Mini-Conference features one principle speaker and several other outstanding combinatorics/graph theorists, as well as some promising young researchers.

Speakers

Featured Speaker

Ronald Graham	University of California, San Diego(UCSD)
Daniela Kühn	University of Birmingham, UK
Deryk Osthus	University of Birmingham, UK

One-hour Speakers

Asaf Shapira	Tel-Aviv University
Wenan Zang	University of Hong Kong
Brendan Nagle	University of South Florida

Half-hour Speakers

Jessica McDonald	Auburn University
Louis DeBiasio	Miami University, Ohio
Giorgis Petridis	University of Georgia
Clifford Smyth	University of North Carolina Greensboro

Schedule

April 22th	Speakers	Title
12:30-12:55pm	Jessica McDonald	List Edge-colouring Graphs with Restricted Odd Cycles
1:00pm-1:50pm	Ronald Graham	The Combinatorics of Solving Linear Equations
2:00pm-2:50pm	Brendan Nagle	An Algorithmic Hypergraph Regularity Lemma
2:50pm-3:20pm	**Break**	
3:20pm-4:10pm	Daniela Kühn	The existence of designs --beyond quasirandomness
4:15pm-4:40pm	Giorgis Petridis	Expansion of polynomials in prime order finite fields

April 23th	Speakers	Title
8:30am-8:55am	Louis DeBiasio	Infinite Graph-Ramsey Theory
9:00am-9:50am	Deryk Osthus	A blow-up lemma for approximate decompositions and the tree packing conjecture
9:55am-10:45am	Wenan Zang	Recent Advances in Polyhedral Combinatorics

10:50am-11:40am	Asaf Shapira	Removal Lemmas with Polynomial Bounds
11:45am-12:10pm	Clifford Smyth	Combinatorial formulas for restricted Stirling and Lah number matrices and their inverses

Abstracts

List Edge-colouring Graphs with Restricted Odd Cycles

Jessica McDonald

In this talk we will look at the class of simple graphs G^* for which every pair of distinct odd cycles intersect in at most one edge. We will give a structural characterization of these graphs, and prove that they satisfy the list-edge-coloring conjecture.

We will also talk about a stronger result concerning kernel-perfect orientations in $L(G)$, which has implications for two different generalizations of list-edge-colouring.

Joint work with Greg Puleo.

The Combinatorics of Solving Linear Equations

Ronald Graham

One of the fundamental topics in combinatorics involves deciding whether some given linear equation has solutions with all its variables lying in some restricted set, and if so, estimating how many such solutions there are. In this talk, we will describe some of the old and new results in this area, as well as discuss a number of unsolved problems.

An Algorithmic Hypergraph Regularity Lemma

Brendan Nagle

Szemerédi's Regularity Lemma is a powerful tool in graph theory. It asserts that all large graphs G admit a bounded partition of $E(G)$, most classes of which are bipartite subgraphs with uniformly distributed edges. The original proof of this result was non-constructive. A constructive proof was given by Alon, Duke, Lefmann, Rödl and Yuster, which allows one to efficiently construct a regular partition for any large graph.

Szemerédi's Regularity Lemma was extended to hypergraphs by various authors. Frankl and Rödl gave one such extension to 3-uniform hypergraphs, and Rödl and Skokan extended this result to k-uniform hypergraphs. W.T. Gowers gave another such extension. Similarly to the graph case, all of these proofs are non-constructive. In this talk, we discuss an efficient algorithmic version of the Hypergraph Regularity Lemma for k-uniform hypergraphs.

The existence of designs
——beyond quasirandomness

Daniela Kühn

The Existence conjecture for combinatorial designs has its roots in the 19th century, and was recently proved by Peter Keevash. We give a new proof of this result, based on the method of iterative absorption. In fact, a "regularity boosting technique" allows us to extend our main decomposition result beyond the quasirandom setting and thus to generalise the results of Keevash. In particular, we obtain a resilience version and version for hypergraphs of large minimum degree.

In this talk, I will present our new results, give a brief outline of the history of the Existence conjecture and provide an overview of our methods.

This is joint work with Stefan Glock, Allan Lo and Deryk Osthus.

Expansion of polynomials in prime order finite fields

Giorgis Petridis

Given a polynomial f in several variables and a set A we denote by $f(A)$ the range of f when all variables are taken to be elements of A. We say that f *expands in the finite field setting* if there exist positive constants δ and c such that for all primes p and all sets A in $\mathbb{F}_p$ (the finite field with p elements) we have $|f(A)| \geq c \min\{p, |A|^{1+\delta}\}$.

We present a historical overview, starting at the first wave of results originating by a breakthrough paper of Bourgain, Katz, and Tao, continuing to the second wave of results originating by a more recent paper of Rudnev, and, if time permits, end by reporting recent progress.

Infinite Graph-Ramsey Theory

Louis DeBiasio

Ramsey's theorem guarantees a monochromatic copy of any countably infinite graph G in any r-coloring of the edges of the complete graph $K_{\mathbb{N}}$. It is natural to wonder how "large" of a monochromatic copy of G we can find with respect to some measure–for instance, the (upper) density of the vertex set of G in $\mathbb{N}$. Unlike finite graph-Ramsey theory, where this question has been studied extensively, the infinite version has been mostly overlooked.

Erdös and Galvin proved that in every 2-coloring of $K_{\mathbb{N}}$, there exists a monochromatic path whose vertex set has upper density at least $2/3$, but it is not possible to do better than $8/9$. They also showed that there exists a monochromatic path P such that for infinitely many n, the set $\{1, 2, ..., n\}$ contains the first $\frac{n}{3+\sqrt{3}}$ vertices of P, but it is not possible to do better than $2n/3$. We improve both results, in the former case achieving an upper density at least $3/4$ and in the latter case obtaining a tight bound of $2/3$. Inspired by this, we consider infinite analogs of well-known finite results on directed paths, trees (connected subgraphs), and graphs of bounded maximum degree.

Joint work with Paul McKenney.

A blow-up lemma for approximate decompositions and the tree packing conjecture

Deryk Osthus

We develop a new tool for constructing approximate decompositions of dense quasirandom graphs into bounded degree graphs. Our result can be viewed as an extension of the classical blow-up lemma of Komlos, Sarkozy and Szemeredi to the setting of approximate decompositions. I will discuss this tool and some of its applications.

In particular, I will discuss applications to the tree packing conjecture of Gyarfas and Lehel: this conjecture asks for a decomposition of a complete graph into a suitable given collection of trees.

Joint work with Felix Joos, Jaehoon Kim, Daniela Kühn and Mykhaylo Tyomkyn.

Recent Advances in Polyhedral Combinatorics

Wenan Zang

Combinatorial optimization searches for an optimal object in a finite collection; typically the collection has a concise representation while the number of objects is huge. Polyhedral and linear programming techniques have proved to be very powerful and successful in tackling various combinatorial optimization problems, and the end products of these methods are often integral polyhedra or min-max relations. This area of combinatorial optimization is called *polyhedral combinatorics*.

In this talk I shall give a brief survey of our recent results on polyhedral combinatorics, including a tournament ranking with no errors, a polyhedral description of kernels, and a characterization of the box-totally dual integral (box-TDI) matching polytope.

Removal Lemmas with Polynomial Bounds

Asaf Shapira

Addressing a problem of Alon and Fox, we prove new sufficient and necessary criteria, guaranteeing that a graph property admits a removal lemma with a polynomial bound. Although both are simple combinatorial criteria, they imply almost all prior positive and negative results of this type, as well as many new ones. In particular, we show that every semi-algebraic graph property admits a polynomially bounded removal lemma. This confirms a conjecture of Alon.

Joint work with L. Gishboliner.

Combinatorial formulas for restricted Stirling and Lah number matrices and their inverses

Clifford Smyth

Given a set R of natural numbers let $S(n, k, R)$ be the restricted Stirling number of the second kind: the number of ways of partitioning a set of size n into k non-empty subsets with the sizes of these subsets restricted to lie in R. Let $S(R)$ be the matrix with $S(n, k, R)$ in its (n, k) entry. If R contains 1, $S(R)$ has an inverse $T(R)$ with integer entries. We find that for many R the entries $T(n, k, R)$ of $T(R)$ are expressible (up to sign) as the cardinalities of explicitly defined sets of trees and forests. For example this is the case when R has no exposed odds, i.e. R contains 1 and 2 and R never contains an odd number n greater than 1 without also containing $n + 1$ and $n - 1$. We have many similar results for restricted Stirling numbers of the first kind (partitions into cycles) and Lah numbers (partitions into ordered lists). Our proofs depend in part on two combinatorial interpretations of the coefficients of the compositional inverse of a power series expressed in terms of trees.

This is joint work with David Galvin of the University of Notre Dame and John Engbers of Marquette University.