

SHORT VITA
Frank J. Hall
December 2022

Education

- Ph.D. North Carolina State University, Raleigh
North Carolina, December, 1973
- M.S. University of Houston, August, 1967
- B.A. St. Mary's University, San Antonio, Texas, May, 1965

Professional Experience

- Jan 1, 2020 Professor Emeritus,
Georgia State University
- 1988-2019 Professor of Mathematics,
Georgia State University
- 1982-1988 Associate Professor of Mathematics,
Georgia State University
- 1978-1982 Assistant Professor of Mathematics,
Georgia State University
- 1974-1978 Assistant Professor, Pembroke State University,
North Carolina
- 1973-1974 Instructor, North Carolina State University
- 1970-1973 Teaching Assistant, North Carolina State University
- 1969-1970 Mathematics Instructor, Kaduna Polytechnic, Kaduna Nigeria,
as member of the United States Peace Corps
- 1967-1968 Teaching Assistant, University of Houston

Publications

1. Generalized inverses of the fundamental bordered matrix used in linear estimation, *Sankhya*, 37A(1975), 428-438, with C.D. Meyer, Jr.
2. Generalized inverses of a bordered matrix of operators, *SIAM J. Appl. Math.*, 29(1975), 152-163.
3. On the independence of blocks of generalized inverses of bordered matrices, *Lin. Alg. and Appl.*, 14(1976), 53-61.
4. Further results on generalized inverses of partitioned matrices, *SIAM J. Appl. Math.*, 30(1976), 617-624, with R.E. Hartwig.
5. On general bordered matrices, *Bull. of the Calcutta Math. Soc.*, 70(1978), 395-403.
6. Pseudo-similarity for matrices over a field, *Proc. of Amer. Math. Soc.*, 71(1978), 6-10, with R.E. Hartwig.
7. A note on Pseudo-similarity of matrices, *The Journal of the Industrial Math. Soc.*, 28(1978), 25-36, with R.E. Hartwig and I.J. Katz.
8. Further results on integral generalized inverses of integral matrices, *Lin. and Multilinear Alg.*, 6(1978), 233-241, with D.R. Batigne and I.J. Katz.
9. The Moore-Penrose inverse of particular bordered matrices, *The Journal of the Australian Math. Soc.*, 27(1979), 467-478.
10. On ranks of integral generalized inverses of integral matrices, *Lin. and Multilinear Alg.*, 7(1979), 73-85, with I.J. Katz.
11. More on integral generalized inverses of integral matrices, *Lin. and Multilinear Alg.*, 9(1980), 201-209, with I.J. Katz.
12. Algebraic properties of governing matrices used in Cesaro-Neumann iterations, *Revue Roumaine de Mathematiques*, 26(1981), 959-978, with R.E. Hartwig.
13. Nonnegative integral generalized inverses, *Lin. Alg. and Appl.*, 39(1981), 23-39, with I.J. Katz.
14. Integer generalized inverses of incidence matrices, *Lin. Alg. and Appl.*, 39(1981), 247-258, with J.H. Bevis and I.J. Katz.
15. Applications of the Drazin Inverse to Cesaro-Neumann iterations, in *Recent Applications of Generalized Inverses*, edited by S.L. Campbell, Pitman Advanced Publishing Program, No. 66, 1982, 145-195, with R.E. Hartwig.

16. Some classes of integral matrices, *Lin. Alg. and Appl.*, 48(1982), 473-483, with J.H. Bevis.
17. Pseudo-similarity and partial unit regularity, *Czechoslovak Math. J.*, 33(1983), 361-372, with R.E. Hartwig, I. J. Katz and M. Newman.
18. Monotonicity of integral matrices, *Math. Japonica*, 28(1983), 501-508, with J.H. Bevis.
19. Block striped and block nested matrices, in Contemporary Mathematics, Linear Algebra and Its Role in Systems Theory, edited by R.A. Brualdi, et al, Amer. Math. Soc., Vol. 47, 1985, 177-201, with R.E. Hartwig and I.J. Katz.
20. Pseudo-consimilarity and semi-consimilarity of complex matrices, *Lin. Alg. and Appl.*, 90 (1987), 73-80, with J.H. Bevis and R.E. Hartwig.
21. Consimilarity and the matrix equation $AX-XB=C$, in Current Trends In Matrix Theory, edited by F. Uhlig and R. Grone, North-Holland, 1987, 51-64, with J.H. Bevis and R.E. Hartwig.
22. The Drazin inverse of a semi-linear transformation and its matrix representation, *Lin. Alg. and Appl.*, 97 (1987), 229-242, with J.H. Bevis and R.E. Hartwig.
23. The matrix equation $AX-XB=C$ and its special cases, *SIAM J. Matrix Anal. Appl.*, 9(1988), 348-359, with J.H. Bevis and R.E. Hartwig.
24. Conpseudosimilarity and consemsimilarity over a division ring, *Lin. Alg. and Appl.*, 136 (1990), 181-188, with J.H. Bevis and R.E. Hartwig.
25. Integer LU- Factorizations, *Lin. Alg. and Appl.*, 150 (1991), 267-285, with J.H. Bevis.
26. LDL^T - factorizations of adjacency matrices where D is block diagonal, *Lin. Alg. and Appl.*, V. 162-164 (1992), 651-662, with J.H. Bevis.
27. Nested range conditions for LU-factorizations of integer matrices, *Lin. Alg. and Appl.*, 172(1992), 97-108, with J.H. Bevis.
28. Self-inverse sign patterns, in IMA Volumes in Mathematics and Its Applications, Vol. 50, 1993, 245-256, Springer-Verlag publisher, with C.A. Eschenbach and C.R. Johnson.
29. Eigenvalue frequency and consistent sign pattern matrices, *Czechoslovak Mathematical Journal*, 44(1994), 461-479, with C.A. Eschenbach and Z. Li.
30. Sign pattern matrices and generalized inverses, *Lin. Alg. and Appl.*, 211(1994), 53-66, with C.A. Eschenbach and Z. Li.

31. On the period and base of a sign pattern matrix, *Lin. Alg. and Appl.*, v. 212-213(1994), 101-120, with C.A. Eschenbach and Z. Li.
32. The structure of nonnegative cyclic matrices, *Lin. and Multilinear Alg.*, 41(1996), 23-33, with C.A. Eschenbach and Z. Li.
33. Some sign patterns that allow a real inverse pair B and B^{-1} , *Lin. Alg. and Appl.*, 252(1997), 299-321, with C.A. Eschenbach and Z. Li.
34. Sign patterns of nonnegative normal matrices, *Lin. Alg. and Appl.*, 254(1997), 335-354, with Z. Li, and F. Zhang.
35. The graphs of the unambiguous entries in the product of two (+,-) sign pattern matrices, *Lin. Alg. and Appl.*, 260(1997), 95-118, with C.A. Eschenbach, Z. Li, and C.R. Johnson.
36. From real to complex sign pattern matrices, *The Bulletin of the Australian Mathematical Society*, 57(1998), 159-172, with C.A. Eschenbach and Z. Li.
37. When does the inverse have the same sign pattern as the transpose?, *Czechoslovak Mathematical Journal*, 49(124)(1999), 255-275, with C.A. Eschenbach, D. Harrell, and Z. Li.
38. Sign patterns of idempotent matrices, *Journal of Korean Mathematical Society*, 36(1999), 469-487, with Z. Li.
39. Eigenvalue distribution of certain ray patterns, *Czechoslovak Mathematical Journal*, 50(125)(2000), 749-762, with C.A. Eschenbach and Z. Li.
40. On almost regular tournament matrices, *Lin. Alg. and Appl.*, 306(2000), 103-121, with J. Weaver et al.
41. Properties of tournaments among well matched players, *The American Mathematical Monthly*, 107(2000), 881-892, with J. Weaver et al.
42. Isomorphisms involving reversing arcs of digraphs, *The Journal of Combinatorial Mathematics and Combinatorial Computing*, 36(2001), 155-160, with G.Chen, A. Kezdy, Z. Li, and H. Zhou.
43. Symmetric sign pattern matrices that require unique inertia, *Lin. Alg. and Appl.*, 338(2001), 153-169, with Z. Li and D. Wang.
44. Inertia sets of symmetric sign pattern matrices, *Numerical Math. J. of Chinese Universities*, 10(2001), 226-240, with Z. Li.
45. Irreducible powerful ray pattern matrices, *Lin. Alg. and Appl.*, 342(2002), 47-58, with Z. Li and J. Stuart.

46. On ranks of matrices associated with trees, *Graphs and Combinatorics*, 19(2003), 323-334, with G. Chen, Z. Li, and B. Wei.
47. Sign patterns, inverses, and generalized inverses – a brief survey, *Numerical Math. J. of Chinese Universities*, 12(Supplement) (2003), 8-11, with Z. Li.
48. On ranks of matrices associated with trees, *Numerical Math. J. of Chinese Universities*, 12(Supplement) (2003), 76-79, with Z. Li.
49. From Boolean to sign pattern matrices, *Lin. Alg. and Appl.*, 393(2004), 233-251, with Z. Li and B. Rao.
50. An interlacing result for Normalized Laplacians, *SIAM J. on Discrete Mathematics*, 18(2004), 353-361, with G. Chen et al.
51. Reducible powerful ray pattern matrices, *Lin. Alg. and Appl.*, 399(2005), 125-140, with Z. Li and J. Stuart.
52. Rational realizations of the minimum rank of a sign pattern matrix, *Lin. Alg. and Appl.*, 409(2005), 111-125, with M. Arav, S. Koyuncu, Z. Li, and B. Rao.
53. Sign pattern matrices, in the Handbook of Linear Algebra, published by Chapman & Hall/CRC, Chapter 33 (invited), 2007, with Z. Li.
54. Inherited LU-factorizations, *Lin. Alg. and Appl.*, 427(2007), 26-41, with M. Arav and J. Bevis.
55. A Cauchy-Schwarz inequality for triples of vectors, *Mathematical Inequalities and Applications journal*, 11(2008), 629-634, with M. Arav and Z. Li.
56. Interlacing results on matrices associated with graphs, *The Journal of Combinatorial Mathematics and Combinatorial Computing*, 68(2009), 113-127, with K. Patel and M. Stewart.
57. Sign patterns that require almost unique rank, *Lin. Alg. and Appl.*, 430(2009), 7-16, with M. Arav, Z. Li, A. Merid, and Y. Gao.
58. Rational solutions of certain matrix equations, *Lin. Alg. and Appl.*, 430(2009), 660-663, with M. Arav, Z. Li, and B. Rao.
59. ZPC matrices and zero cycles, *International Journal of Combinatorics*, (2009), Article ID 520923, 5 pages, with M. Arav, Z. Li, and B. Rao.
60. Sign patterns that allow eventual positivity, *Electronic Journal of Linear Algebra*, 19(2010), 108-120, with A. Berman et al. (rigorously refereed)

61. Spectrally arbitrary tree sign patterns of order 4, *Electronic Journal of Linear Algebra*, 20(2010), 180-197, with M. Arav, K. Kaphle, Z. Li, and N. Manzagol. (rigorously refereed)
62. Some inheritance properties for complementary basic matrices, *Lin. Alg. and Appl.*, 433(2010), 2060-2069, with M. Fiedler.
63. G-matrices, *Lin. Alg. and Appl.*, 436(2012), 731-741, with M. Fiedler.
64. A note on permanents and generalized complementary basic matrices, *Lin. Alg. and Appl.*, 436(2012), 3553-3569, with M. Fiedler.
65. Gersgorin discs revisited, *Lin. Alg. and Appl.*, 438(2013), 598-603, with M. Fiedler and R. Marsli.
66. Geometric multiplicities and Gersgorin discs, *The American Mathematical Monthly*, 120 (2013), 452-455, with R. Marsli.
67. Sign patterns with minimum rank 2 and upper bounds on minimum ranks, *Linear and Multilinear Algebra*, 61(2013), 895-908, with Z. Li, Y. Gao, M. Arav, F. Gong, W. Gao, and H. van der Holst.
68. Sign pattern matrices, in the *Handbook of Linear Algebra*, 2nd edition, published by Chapman & Hall/CRC, Chapter 42 (invited), 2013, with Z. Li.
69. Some graph theoretic properties of generalized complementary basic matrices, *Lin. Alg. and Appl.*, 438(2013), 3365-3374, with M. Fiedler.
70. Combinatorial aspects of generalized complementary basic matrices, *Central European Journal of Mathematics*, 11(2013), 2186-2196, with M. Fiedler.
71. Further results on Gersgorin discs, *Lin. Alg. and Appl.*, 439(2013), 189-195, with R. Marsli.
72. The inertia set of a signed graph, *Lin. Alg. and Appl.*, 439(2013), 1506-1529, with M. Arav, Z. Li, and H. van der Holst.
73. Some refinements of Gersgorin discs, *International Journal of Algebra*, 7(2013), 573-580, with R. Marsli.
74. Factorizable matrices, *Special Matrices journal*, 1(2013), 3-9, with M. Fiedler.
75. Dense alternating sign matrices and extensions, *Lin. Alg. and Appl.*, 444(2014), 219-226, with M. Fiedler and M. Stroeve.
76. The minimum rank of a sign pattern matrix with a 1-separation, *Lin. Alg. and Appl.*, 448(2014), 205-216, with M. Arav, Z. Li, H. van der Holst, L. Zhang, and W. Zhou.

77. Some new inequalities on geometric multiplicities and Gersgorin discs, *International Journal of Algebra*, 8(2014), 135-147, with R. Marsli.
78. Permanents, determinants, and generalized complementary basic matrices, *Operators and Matrices*, 8(2014), 1041-1051, with M. Fiedler and M. Stroev.
79. Max algebraic complementary matrices, *Lin. Alg. and Appl.*, 457(2014), 287-292, with M. Fiedler.
80. A new characterization of generalized complementary basic matrices, *Special Matrices journal*, 2(2014), 125-130, with M. Fiedler.
81. Ranks of alternating sign matrices and their sign patterns, *Lin. Alg. and Appl.*, 471(2015), 109-121, with M. Fiedler, W. Gao, G. Jing, Z. Li, and M. Stroev.
82. A note on strongly dense matrices, *European Journal of Mathematics*, 1(2015), 721-730, with M. Fiedler.
83. Minimum ranks of sign patterns via sign vectors and duality, *Electronic Linear Algebra journal*, 30(2015), 360-371, with M. Arav, Z. Li, H. van der Holst, J. Sinkovic, and L. Zhang.
84. Signed graphs whose signed Colin de Verdiere parameter is two, *J. of Comb.Theory B*, 116(2016), 440-455, with M. Arav, Z. Li, and H. van der Holst.
85. Miroslav Fiedler (7.4.1926-20.11.2015), *Czechoslovak Math. J.*, 66(2016), 585-590, with T. Markham, M. Rozložnik, and J. Stuart.
86. G-matrices, J-orthogonal matrices, and their sign patterns, *Czechoslovak Math J.*, 66(2016), 653-670, with M. Rozložnik.
87. Linear preservers of row-dense matrices, *Czechoslovak Math J.*, 66(2016), 847-858, with A. Armandnejad and S. Motlaghian.
88. Linear preservers of Hadamard majorization, *Electronic Journal of Linear Algebra*, 31(2016), 593-609, with A. Armandnejad and S. Motlaghian.
89. On the location of eigenvalues of real matrices, *Electronic Journal of Linear Algebra*, 32(2017), 357-364, with R. Marsli.
90. Sign patterns of J-orthogonal matrices, *Special Matrices journal*, 5(2017), 225-241, with Z. Li, C. Parnass, and M. Rozložnik.
91. Strong linear preservers of dense matrices, *Bull. Iranian Math. Soc.*, 44(2018), 969-976, with A. Armandnejad and S. Motlaghian.
92. Topological properties of J-orthogonal matrices, *Linear Multilinear Algebra*, 66(2018),

- 2524-2533, with A. Armandnejad and S. Motlaghian.
93. On discriminant matrices, *Lin. Alg. and Appl.*, 554(2018), 316-328, with M. Stroeve.
 94. On the location of eigenvalues of real constant row-sum matrices, *Bull. Korean Math. Soc.*, 55(2018), 1691-1701, with R. Marsli.
 95. On bounding the eigenvalues of matrices with constant row-sums, *Linear Multilinear Algebra*, 67(2019), 672-684, with R. Marsli.
 96. Rank conditions for sign pattern matrices that allow diagonalizability, *Discrete Math.*, 343(2020), 111798, with Z. Li et al.
 97. Equivalence classes of e-matrices and associated eigenvalue localization regions, *Linear Multilinear Algebra*, 68(2020), 915-930, with R. Marsli.
 98. Inclusion regions and bounds for the eigenvalues of matrices with a known eigenpair, *Spec. Matrices*, 8(2020), 204-220, with R. Marsli.
 99. Two-connected signed graphs with maximum nullity at most two, *Lin. Alg. and Appl.*, 611(2021), 82-93, with M. Arav, Z. Li, and H. van der Holst.
 100. Topological properties of J-orthogonal matrices, Part II, *Linear Multilinear Algebra*, 69(2021), 438-447, with A. Armandnejad and S. Motlaghian.
 101. Some properties of ergodicity coefficients with applications in spectral graph theory, *Linear Multilinear Algebra*, 70(2022), 1886-1906, with R. Marsli.
 102. A note on some classes of G-matrices, *Operators and Matrices*, 16(2022), 251-263, with A. Armandnejad and S. Motlaghian.
 103. Two n by n G-classes of matrices having finite intersection, *Special Matrices*, 11(2023), 20220178, with S. Golshan and A. Armandnejad.

Articles 37, 43, 50, 52, 56, 57, 60, 61, 65, 66, 67, 71, 73, 75, 76, 77, 78, 81, 87, 88, 90, 91, 92, 96, 100, 103 were published with students.

Papers in Process

4 by 4 irreducible sign patterns that require four distinct eigenvalues, with Y. Gao, Z. Li, V. Bailey, P. Kim, submitted for publication.

Eigenvalues of generalized complementary basic matrices, with Z. Li, M. Rozloznik, M. Stewart.

Similarity via transversal intersection of manifolds, with M. Arav, Z. Li, H. van der Holst, et al.

A new algorithm for Boolean matrix factorization, with M. Arav et al.

Boolean matrix factorization in genetic data analysis, with M. Arav et al.

The normalized Laplacian of a signed simple graph, with K. Amin and M. Stewart.

Involutory solutions to $AX = XA^{-1}$ over a closed field, with R.E. Hartwig.

Masters Theses

The Drazin Inverse, Laurence Grodd, completed 3/81, thesis director.

The Inverse Transpose Problem, Deborah Harrell, completed 3/96, thesis committee.

Fisher's Linear Discriminant Analyses for Pairwise Correlated Data, Sharon Annan, completed 4/97, thesis committee.

Possible inertias of matrices with a specified sign pattern, Di Wang, completed 8/98, thesis committee.

Some eigenvalue results for certain matrices associated with graphs, Kinnari Patel, completed 4/04, thesis co-director.

Minimum rank 2 sign pattern matrices, Emmanuel Des-Bordes, completed 4/04, thesis committee.

Characterization of bar visibility graphs with bounded lengths for trees, Ken Keating, completed 6/04, thesis committee.

Rational realizations of the minimum rank of a sign pattern matrix, Selcuk Koyuncu, completed 11/05, thesis co-director.

On some aspects of the differential operator, Panakkal Mathew, completed 5/06, thesis committee.

Spectrally arbitrary tree sign pattern matrices, Krishna Kaphle, completed 11/06, thesis committee.

Spectrally arbitrary and inertially arbitrary sign pattern matrices, Nilay Demir, completed 3/07, thesis director.

The square root function of a matrix, Crystal Gordon, completed 4/07, thesis co-director.

Singular value decomposition and principal component analysis in image processing, Wasuta Renkjunong, completed 6/07, thesis committee.

The exponential function of matrices, Nathalie Smalls, completed 11/07, thesis committee.

Sign patterns that require almost unique rank, Assefa Merid, completed 4/08, thesis co-director.

Intersections of longest paths and cycles, Thomas Hippchen, completed 4/08, thesis committee.

Riccati equations in optimal control theory, James Bellon, completed 4/08, thesis committee.

Singular value decomposition in image noise filtering and reconstruction, Tsegaselassie Workalemahu, completed 5/08, thesis committee.

Noetherian filtrations and finite intersection algebras, Sara Malec, completed 7/08, thesis committee.

Factorization of quasiseparable matrices, Paul Johnson, completed 11/08, thesis committee.

Examination of initialization techniques for nonnegative matrix factorization, John Frederic, completed 11/08, thesis committee.

Applications of linear algebra to information retrieval, Jhansi Vasireddy, completed 3/09, thesis director.

Method for improving the efficiency of image super-resolution algorithms based on Kalman filters, William Dobson, completed 12/09, thesis committee.

On the 4 by 4 irreducible sign pattern matrices that require four distinct eigenvalues, Paul Kim, completed 7/11, thesis co-director.

Geometric multiplicities and Gersgorin discs, Rachid Marsli, completed 11/12, thesis director.

The topology and algebraic functions on affine algebraic sets over an arbitrary field, Anthony Preslicka, completed 11/12, thesis committee.

The minimum rank of sign pattern matrices with a 1-separation, Wenyan Zhou, completed 7/13, thesis committee.

Leslie Matrices, Darci Chambers, in process, thesis director.

Sign patterns of J-orthogonal matrices, Caroline Parnass, completed 11/17, thesis co-director with Z. Li.

PhD Theses:

Intersection algebras and pointed rational cones, Sara Malec, completed 6/13, thesis committee.

HISTs, Halin graphs, and long cycles in 3-connected graphs, Songling Shan, completed 4/15, thesis committee.

Minimum ranks and refined inertias of sign pattern matrices, Wei Gao, completed 5/16, thesis committee.

Signed graphs with maximum nullity at most two, F. Scott Dahlgren, thesis defense Feb 25, 2022, thesis committee.

New extensions and applications of the Gersgorin theory, Rachid Marsli, completed 7/15, thesis director. Present position: Assistant Professor, King Fahd University of Petroleum & Minerals, Dhahran, Saudi Arabia.

Some results on generalized complementary basic matrices and dense alternating sign matrices, Mike Stroeve, completed 4/16, thesis director. Present position: Academic Professional, GSU.

Row dense matrices, dense matrices, Hadamard majorization, and their linear preservers, Sara Motlaghian, thesis co-director with A. Armandnejad, Vali-E-Asr University of Rafsanjan, Iran, thesis defense July 17, 2017.

Sign patterns that allow diagonalizability, Christopher Zagrodny, completed 12/18, thesis co-director with Z. Li.

Topological properties of J -orthogonal matrices, Sara Motlaghian, completed 11/20, thesis director. Present position: Postdoctoral Research Assistant, Trends Center, GSU.

Masters Research Papers

Irreducible normalized maximal SNS patterns, Kim Hunter, completed 8/98, director.

Sign pattern matrices that require exactly two real eigenvalues, Eugenia Rubinshtein, completed 6/98, reader.

Sign pattern matrices that require distinct eigenvalues, Laura Harris, completed 12/98, reader.

Applications of algebra in coding theory, Chris Smith, completed 8/2000, reader.