Spring 2010 Polynomials Homework-Assignment 5 **Name:** ____________________

Write-up your solution carefully including all the details of the proof. Due November 4.

Please staple your assignment.

(1) (5 points)
Compute the apolar derivative of $f(z) = z^4 + 2z + 1$ with respect to $\xi = 2i$.

(2) (5 points) Find an apolar polynomial for $f(z) = z^4 + 3z^2 + 3$.

(3) (5 points) Determine whether or not $f(z) = z^3 + 2z + 2$ and $g(z) = z^3 + z^2 + 1$ have a common root.

(4) (5 points)
Let $f(z) = z^3 + 6z^2 - 3z + 3$. Show that at least one root of f has absolute value less or equal to 2. (hint: consider a polynomial $g = z^3 + z^2 + az$ apolar with f).

(5) (5 points)(graduate students) If $f(z) = z^3 + 3z^2 + 9z + a = 0$, $a \in \mathbf{R}$ has a complex nonreal root w, then $\mid Im(w) \mid > \sqrt{2}$.