Spring 2010 Polynomials Homework-Assignment 1 **Name:** _____

Write-up your solution carefully including all the details of the proof. Due Thursday, September 9.

Please staple your assignment.

1. **(5 points) Consider the equation $ax^2 + bx + c = 0$ with real coefficients and no real solutions. Assume that $a + b + c > 0$. What sign does the expression $a - b + c$ have? Prove it.**

2. **(5 points) If α and β are the roots of $ax^2 + bx + c = 0$, write down the equation of degree two (and coefficients expressed in terms of a, b, c) that has roots $2\alpha + 3\beta$ and $2\beta + 3\alpha$.**

3. **(5 points) Find the four roots of the equation $z^4 + 4$ and use them to factor $z^4 + 4$ as a product of two quadratic polynomials with integer coefficients.**

4. **(5 points) Show that $\mid z \mid < 1$ implies that $\mid Im(1 - \overline{z} + z^2) \mid < 3$.**

5. **(5 points)(graduate students) Consider $a, b, c, m \in \mathbf{R}$ such that $m > 1$. If**

$$\frac{a}{m+1} + \frac{b}{m} + \frac{c}{m-1} = 0,$$

show that $b^2 - 4ac \geq 0$.